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SEPARATION OF HYDROGEN ISOTOPES BY DIFFUSION THROUGH PALLADIUM

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Introductory.—The diffusion method for separating the hydrogen isotopes suggested by Hertz[1] deals with the molecules H^IH^I and H^IH^{II} which have a mass ratio of 1.5. Any process dealing with the separate atoms should be much more efficient since the mass ratio would be 2. An example of such a process is the diffusion of hydrogen through palladium.

In such diffusion processes in solids, it is known[2] that generally an exponential factor occurs in which there enters a work function that may well be different from H^I than for H^{II} due to the difference in zero point energy of the hydrogen atoms in the palladium lattice; this exponential factor would thus also favor the preferential diffusion of the lighter isotope. It is not possible to make any quantitative predictions about the importance of this factor. Furthermore, depending on the experimental conditions, the speed of the process may be determined by either the rate of diffusion in the solid or the transition gas-solid and solid-gas transition.

Although we do not anticipate that the method suggested here will compete with the electrolytic method of concentration[3] yet an investigation of it seems of considerable theoretical importance especially with regard to the mechanism of diffusion in solids. In special cases, of course, the method might prove useful to obtain in a single step a rather large increase in concentration of the heavier isotope in a small sample of hydrogen.

Theoretical Considerations.—A rough estimate of the concentration to be expected has been made under the following simplifying assumptions:

1. The concentration of migration H atoms (or ions) in palladium, in equilibrium with the H_2 gas, is proportional to the square root of the pressure.

2. The ratio of concentration of the isotopes in Pd is the same as in the gas.

3. If the gas at the low pressure side of the Pd is pumped off with a fast high vacuum pump, the rate of diffusion is proportional to the H concentration on the high pressure side, i.e., to the square root of the pressure. Let the velocity constants for H^I and H^{II} be k^I and k^{II} (our reasoning holds for both cases: speed determined by diffusion process or by transition solid-gas), respectively. With these assumptions it can be shown that

$$\frac{p_f^{II}}{p_o^{II}} = \left[\frac{p_f^{I}}{p_o^{I}}\right]^{\frac{k^{II}}{k^{I}}}$$

where the subscripts f and o refer to the final and initial states, respectively, and the superscripts refer to the masses of the isotopes. If we put $\frac{k^{II}}{k^{I}} = 0.7$, the inverse ratio of the square roots of the masses, and make $p_f = \frac{1}{100} p_o^{\text{total}}$ (i.e., consider we pump an amount of hydrogen consisting mainly of the lighter isotopes through a heated palladium tube down to 1/100 of its original pressure) then

$$\frac{p_f^{I}}{p_o^{I}} = \frac{1}{100} \text{ and } \frac{p_f^{II}}{p_o^{II}} = \frac{1}{25}.$$

Thus, the relative increase of concentration of the heavier isotope in the remaining gas is four times. The above calculation has been made omitting the exponential factor mentioned above. If this factor is of importance the enrichment should be greater. Our experiments seem to indicate that such is the case since the relative increase in concentration is about ten times. This fact suggests that the increase might be still further improved by working at a lower temperature.

Experimental.—The apparatus used was very simple in principle. About 30 liters of hydrogen were contained in five bulbs arranged in such a way as to allow free circulation of the hydrogen through the system by means of convection. This was done to prevent a localized concentration of the isotopes around the palladium tube. The hydrogen used was obtained by passing water vapor over heated iron; the water was a sample from the Laboratory's supply of "electrolyzed water" and contained about 1 part in 1000 of the heavier isotope.

In this preliminary experiment a single palladium tube heated electrically to a temperature below 400°C. was used. The efficiency of the process may be improved by pumping in stages through several palladium tubes.

The decrease of pressure with time was observed (Fig. 1) and gives support to assumptions 1 and 3.

Changes in the relative concentrations of the isotopes were observed spectroscopically. The problem was simplified by the fact that we were interested in the relative concentration of the heavier isotope before and after the experiment rather than in determining the absolute concentra-

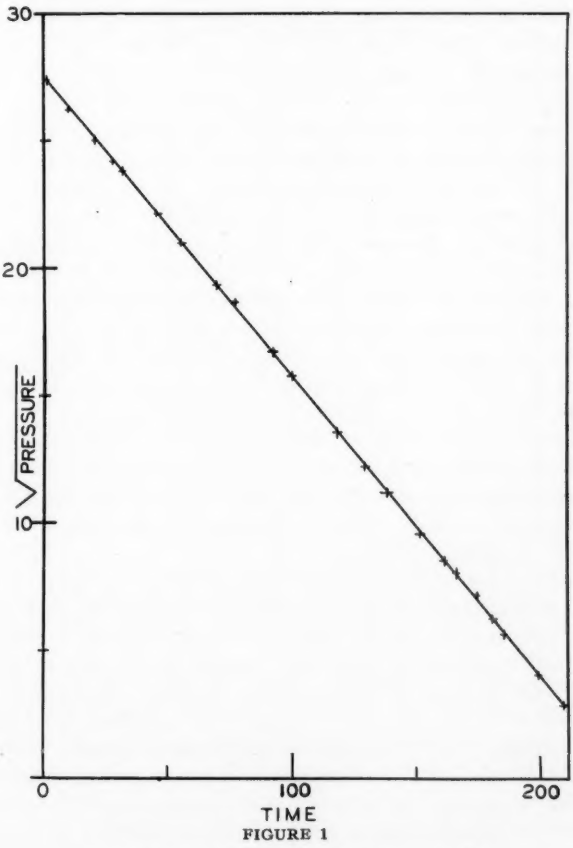

FIGURE 1

Square root of pressure (mm.) *vs.* time (hours).

tion relative to the lighter isotope. Samples of hydrogen were taken from the bulbs and examined

(1) at the beginning of the experiment (750 mm. pressure)
(2) at 39 mm. pressure
(3) at 8 mm. pressure.

The discharge tube was a slightly modified form of one described by Urey.[4] The tube was operated with a current of 0.7 ampere, at 1000 volts drop in potential, the source being a d. c. motor generator set. It was found that after the tube had been flushed several times, with the sample of hydrogen under investigation, it gave fairly reproducible intensities for the isotope lines. Photographs were taken on the 35-foot concave grating with 180,000 lines ruled 30,000 lines to the inch. The lines of the heavier isotope were obtained with exposures of 30 to 100 minutes with tank hydrogen; with the concentrated samples the lines were obtained strongly with exposures of five minutes. Photographs of the heavy isotope lines were obtained with H_α, H_β, H_γ and H_δ in the first order and with H_β and H_α in the second order. For photometric purposes it was found most convenient to use H_α in the first order. With a given sample of hydrogen the procedure was as follows. A photograph was taken with an exposure of five minutes after the tube had been well flushed. This was repeated a second time. Similar exposures were then made through calibrated wire screens. The intensity of the line due to the heavier isotope in the enriched samples, decreased by the screens, was compared with the intensity due to the heavier isotope in the original sample with the aid of microphotometer measurements.

These measurements indicate that the gas was enriched in the heavier isotope from 5 to 8 times when the pressure was reduced from 750 to 39 mm. and a further enrichment of about 1.5 times when the pressure was further reduced to 8 mm. Thus a 100-fold decrease in pressure gave an enrichment of about 10 times.

Summary.—The results presented here seem to substantiate the fact that the diffusion of hydrogen through palladium is through an atomic process. The results further indicate that an activation factor is involved which operates to favor the preferential diffusion of the lighter isotope. This method of separating the hydrogen isotopes thus offers considerable advantage over the usual diffusion methods and has led to a ten-fold increase in the concentration of heavier isotope in a single step. Although this method of concentration cannot compete with the electrolytic one it may well supplement it in special cases.

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[4] Urey, Murphy and Duncan, *Rev. Sci. Inst.*, **3**, 497 (1932).

SUMMARY OF A VARIABLE STAR SURVEY IN AN EXTERNAL GALAXY

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The Clouds of Magellan are in many respects the most satisfactory of external galaxies. Their dimensions are like those of the general run of spiral, spheroidal and irregular nebulae. The Large Cloud should be classed among the giant systems; the Small Cloud among those of average size. Their chief advantage over typical spirals, however, is their relative nearness. This proximity (less than 100,000 light years) permits even moderate telescopes to photograph individually tens of thousands of the brightest stars; and in the near future large reflectors may, under favorable and persistent exposure, photograph in them stars as faint intrinsically as our sun. Although they are galaxies of normal size, the Magellanic Clouds might well be treated as satellites of our own galactic system—or at least as parts of the supergalaxy which includes Milky Way star clouds, globular star clusters and the half dozen external galaxies that lie within a million light years.

At the Harvard Observatory we have just completed an extensive survey of Cepheid variable stars in the Large Cloud. The study is preliminary to a general investigation of the structure of the system. The more important results are summarized in the following paragraphs. The full paper will appear in *Harvard Annals*, Volume **90**, and will include the positions and magnitudes of six hundred newly discovered variable stars.

1. Nearly thirty years ago Miss Leavitt published positions and rough magnitudes of about eight hundred variable stars in the Large Cloud.[1] Subsequently other Harvard observers have determined provisionally the periods and light curves for forty of these variables.[2] During the past five years we have increased the number of known variable stars in the Cloud to 1346. The work has been based on photographs accumulated with the 24-inch Bruce refractor at the Boyden Station, first in Arequipa, Peru, and now at Bloemfontein, South Africa; Miss Sylvia Mussells has been largely responsible for the marking and measuring of the new variables.

2. The material now available is scarcely sufficient for thorough investigation of the periods and light curves of the individual variables. We have therefore supplemented the measures of position with about twenty thousand estimates of magnitudes, referred to international standards,

providing for each variable fairly trustworthy values of medium magnitude and amplitude of variation.

3. The variable stars are scattered throughout the whole area of the Cloud. Since general frequency curves of all stars of high luminosity in the Cloud have already been derived,[3] it has been possible to obtain for the first time a fairly definite idea of the incidence of Cepheid variability among supergiant stars within given intervals of luminosity. Taking all regions of the Cloud together, we find that about one and a half per cent of all the supergiant stars between absolute magnitudes -1 and -4 are Cepheid variables; but along the concentrated axis the proportion rises to approximately four per cent, whereas in sparsely populated parts of the Cloud less than one star in two hundred is a Cepheid variable.

4. The survey of Cepheid variables in the Large Cloud gives the frequency curve of median absolute magnitudes of Cepheids, and therefore, by way of the period-luminosity relation, the frequency curve of the periods. The most numerous periods are between two and three days, and there is no evidence of discontinuity in the luminosity curve for longer periods. When the survey is extended to fainter magnitudes it is probable that the maximum frequency will shift further toward the shorter periods.

5. There is no important correlation between amplitude of variation and luminosity or period. A wide dispersion is found in the amplitudes, whatever the period, values between half a magnitude and two magnitudes appearing for the supergiant Cepheids as well as for variables of absolute magnitude -1.

6. Light absorption by nebulosity within the Cloud is clearly shown for variables in the vicinity of bright gaseous nebulae; and an estimate of the average absorption throughout the whole system will soon be made possible by an investigation of the number and brightness of remote external galaxies that can be seen through the Cloud.

7. Further studies of the variables in the Cloud will include, of course, the determination of periods for many individual stars in special regions, and also the search for classical Cepheids lying beyond the currently recognized bounds of the Cloud. From a study of long exposure photographs, the diameter of the Cloud has been judged to be approximately seven degrees. But new evidence from star counts and especially from discoveries of affiliated clusters, indicates that the actual diameter of the system is more like twelve degrees, and its linear diameter not less than fifteen thousand light years.

[1] *H. A.*, **60**, No. 4 (1908).
[2] *H. B.* 883 (1931).
[3] *H. B.* 881 (1931).

THE TERRESTRIAL ABUNDANCE OF THE PERMANENT GASES

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The recent identification of neon in the nebulae, by forbidden emission lines of Ne III[1] and in certain stars, by absorption lines of Ne II,[2] indicates that it is cosmically an abundant element, comparable with sulphur or phosphorus, if not with carbon. Argon, which has not been detected in stars or nebulae, though its lines are as favorably placed, appears to be much less common. On earth, however, argon is some 500 times as abundant (by volume) as neon—on the reasonable assumption that the whole supply of these gases is to be found in the atmosphere. The general agreement of the terrestrial and cosmical abundances of most elements, and especially of the metals, calls for an explanation of the discrepancy. The obvious explanation is that the Earth has lost practically all of its "initial" neon by escape of atoms into space, as the Moon has lost its atmosphere.

It does not appear to have been noticed that a still more striking case of the same sort is presented by nitrogen. Both stars and nebulae show that this is one of the most abundant of all the elements, seriously exceeded in quantity only by hydrogen, helium and oxygen. On earth it is almost as strongly "atmophile" as the noble gases, and forms but a beggarly 0.02 per cent, by weight, even of the superficial material (outer ten miles) as against 49 per cent of oxygen and from 7.5 to 1.9 per cent for the commoner metals.[3] An amount of nitrogen comparable with these could scarcely be held in combination in the deeper layers; hence the conclusion seems unavoidable that it, too, has escaped.

There remains, of course, the alternative that the abundances derived from spectroscopic study of the reversing layer may not be representative of lower levels. Owing to gravitational separation, the lighter elements may be more concentrated in the upper levels. Hydrogen and helium, for example, may well occur less abundantly in the sub-photospheric layers. But mere gravitational sorting could not produce a marked separation down to any reasonable depth from which the earth may have come, of atoms that differ so little in weight as nitrogen, oxygen and neon. In fact, for the two last-mentioned elements, any hydrostatic separation would act only to increase the discrepancy. The presence of lead and the rare-earths in the reversing layer and chromosphere is additional evidence that the gravitational effect is negligible.

Menzel and Payne[4] have found evidence of non-uniform cosmic distribution of certain elements Hence there is also the possibility that

both nitrogen and neon may be less abundant in the sun than elsewhere. Oxygen, certainly, is abundant in the sun. The atomic lines of nitrogen and neon are, however, not so advantageously placed in the spectrum and the only observable lines of these elements have excitation potentials appreciably higher than those of the observed oxygen lines. Oxygen lines of excitation and laboratory intensity comparable to those of nitrogen do not appear in the Fraunhofer spectrum. The strength of the CN bands is evidence that the abundances of carbon and nitrogen are in no way abnormal for a star of spectral class GO. Neon lines should appear in the flash if the element were as abundant as helium. Their failure to show, in the absence of a satisfactory theory of chromospheric excitation, cannot be definitely interpreted at present, but the upper limit for the abundance would, nevertheless, be quite high.

Moulton, in 1905,[5] suggested that the difference in mean density between the terrestrial and major planets arose from the retention by the latter of "light volatile substances" that had escaped from the former. It has long been recognized that this must be true of hydrogen, and one of us[6] has shown that the great concentration of its heavier isotope on earth, compared with the Sun, may thus be simply explained, but the escape of gases as heavy as N_2 has hitherto not been considered possible. In the present condition of the earth, it is doubtless negligible.

According to Jeans,[7] the rate of escape is given by

$$t_1 = \frac{C^3}{2g^2 a} e^{\frac{3ga}{C^2}} \tag{1}$$

where t_1 is the time that would be required for the complete disappearance of a given gas from the outer atmosphere if the present rate of loss were kept up. C is the mean-square molecular velocity for the gas, a the planet's radius and g its surface gravity. This formula is at best only approximate, but it should lead to values of the correct order of magnitude as long as $3ga/C^2 > 1$. If V is the escape velocity at the surface and t' the time of revolution of a satellite in a circular orbit just outside, $V^2 = 2ga$, $t' = 2\pi\sqrt{a/g}$ so that

$$\frac{t_1}{t'} = \frac{1}{\pi\sqrt{2}} \left(\frac{C}{V}\right)^3 e^{\frac{3}{2}V^2/C^2}. \tag{2}$$

For the earth (and all other bodies of the same mean density) $t' = 1^h24^m$ and we find

V/C	3.0	3.5	4.0	4.5	4.75	5.0
t_1 (years)	1	80	1×10^4	6×10^6	2×10^8	5×10^9

At the earth's surface $V = 1.2 \times 10^4$ cm./sec. For a gas of molecular weight m and temperature T, $C = 1.58 \times 10^4\sqrt{T/m}$. The loss during geological time should be small if $V/C > 5$, that is, if $T < 230\,m$, and practically complete if $V/C < 4.75$ or $T > 260\,m$. Even with a temperature at the top of the atmosphere as high as 400°K., the loss of hydrogen should be small, and of other gases negligible—as Jeans states.

In fact, however, there is practically no hydrogen in our atmosphere, and much less helium than has probably been liberated by the weathering of igneous radioactive rocks during geological time. This difficulty has been particularly discussed by Johntone Stoney and was used by him as an argument against the validity of the classical treatment of the escape problem. Fast-moving molecules must be more common in the upper atmosphere than the kinetic theory predicts. It may be suggested that these rapid motions are due to collisions of the second kind between atoms of helium (for example) and excited atoms of other elements. The invariable presence of the auroral line in the spectrum of the night sky shows that oxygen atoms in the metastable ^1S state are normally present in the upper atmosphere. Such an atom has available energy corresponding to 4.17 electron-volts, or 6.63×10^{-12} ergs. If a collision with a helium atom should discharge this energy, four-fifths of it, or 5.30×10^{-12} ergs, would appear as kinetic energy of the latter, imparting to it a speed of 1.26×10^6 cm./sec.—which exceeds the escape velocity. This process would eliminate only hydrogen and helium. A collision between an excited and a normal oxygen atom could at most impart to each a velocity of 5.02×10^5 cm./sec.—too little for escape.

To lose its neon, a body of the earth's size and mass would have to maintain a surface temperature of 5000°K. for millions of years or 8000° for a few centuries. For nitrogen, the corresponding values are 40 per cent higher. The maintenance of such temperatures is quite inadmissible in an independent body of this size; but the assumed loss may easily have taken place if the earth was formed by ejection of matter from the sun.

Our ignorance of the initial conditions of eruption is too great to permit any detailed numerical discussion. The liberated material, however, must have come from some considerable depth, for the whole amount of metals above the photosphere is of the order of 0.05 gm./cm.^{2}, or 3×10^{21} gm. for the whole surface—less than a millionth part of the earth's mass.[8] The ejected material must therefore have been originally much hotter than the photosphere. Even if it had been concentrated from the start into a body of the earth's high density, it would have lost most, if not all, of its gaseous constituents.[9] Had it been at any time in its career dispersed into separate masses as large as the Moon, and *a fortiori* into planetesimals,[10] the loss would have been total.

The most favorable case for the escape of atoms is from an isothermal

sphere. The energy radiated in time dt, from the surface of a sphere of temperature T is, by Stefan's law $4\pi R^2\sigma T^4 dt$. If the sphere cools uniformly and without shrinking this must be equal to the amount of heat lost by the sphere in cooling through the temperature range dT, or $\frac{4}{3}\pi R^3 \frac{\rho}{m} C_v dT$, where ρ is the mean density and C_v the specific heat at constant volume. C_v will depend on the type of gas and degree of dissociation, but the value should lie within the range $\frac{3}{2}\mathbf{R}$ to $\frac{7}{2}\mathbf{R}$. On setting $C_v = \frac{5}{2}\mathbf{R}$, we have for the condition that the amount of heat radiated shall be equal to that lost by the sphere in cooling

$$\frac{dT}{dt} = \frac{6}{5}\frac{m\sigma T^4}{\rho \mathbf{R} R} = 1.2 \times 10^{-21}\, T^4 \frac{m}{\rho}\frac{R_0}{R}. \qquad (3)$$

$\mathbf{R} = 8.3 \times 10^7$ and $\sigma = 5.7 \times 10^{-5}$. R_0 is the radius of the earth, 7×10^8 cm. The present value of the bracketed factor is about 10. The original values of R and ρ are not known. It is evident, however, that the bracketed factor, under the primitive condition of greater radius and lower density, must have been much higher. Hence, for original temperatures of 5000° or more

$$\frac{dT}{dt} \gg 7 \times 10^{-6} \text{ deg. sec.}^{-1} \text{ or } 200 \text{ deg. yr.}^{-1}. \qquad (4)$$

Lack of perfect conductivity would have caused the surface temperature to fall still more rapidly. Additional heat would have been provided by ionic association, formation of molecules and gravitational contraction, but this could not have been more than from five to ten times the amount of heat provided by simple cooling processes and the argument of rapid temperature fall is still valid. It seems unlikely that a temperature as high as 5000° could have prevailed for more than a few years, a conclusion substantially in accord with that of Jeffrey's. Escape from the surface of the distended planet would, of course, have been facilitated by the smaller value of V, but this could not account for the escape from low levels. The conclusion seems unavoidable that most of the loss occurred during the first few years if not the first few days of the planet's independent existence, with the loss of hydrogen practically immediate.

The outstanding problem appears, therefore, not to account for the loss of the permanent gases from the earth, but to explain why any at all are left. For substances such as water and carbon dioxide, which may enter into the composition of molten magma, no difficulty arises, and the earth's free oxygen may be a product of vegetation during later geologic time.

The argon, nitrogen and neon are, however, probably primitive. The ratio of the molecular weights of the last two is such that if the nitrogen were heavily depleted by a process of escape, near the limiting temperature that permitted it, the neon would disappear altogether. A rapid escape, at a temperature initially much above the limit, but soon lowered by a general cooling of the new-born planet's surface, so that it did not go to the limit, appears competent to explain the facts.

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[3] F. W. Clarke and H. S. Washington, *Ibid.*, **8**, 114 (1922).
[4] Unpublished. Cf., also C. H. Payne, *Zeits für Astrophys.*, **7**, 1 (1933).
[5] *Astrophys. Jour.*, **22**, 176 (1905).
[6] D. H. Menzel, *Publ. Astr. Soc. Pac.*, **44**, 41 (1932).
[7] *The Dynamical Theory of Gases*, 3rd Ed., Cambridge (1921), p. 346.
[8] H. N. Russell, *Astrophys. Jour.*, **70**, 11 (1929).
[9] Jeffreys, *The Earth*, Cambridge (1929), p. 35.
[10] Chamberlin, *The Two Solar Families*, Chicago (1928), p. 168.

ON THE LINEAR DIAMETERS OF 125 LARGE GALAXIES

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1. In progress toward the interpretation of our own galactic system we naturally look for analogous systems among the external galaxies, the size and composition of which may suggest the dimensions and structure of our own system. The Andromeda Nebula may be taken as approximately twelve kiloparsecs in maximum diameter, if no serious alteration becomes necessary in the zero point of the period-luminosity curve for Cepheid variables. Recent investigations of galactic rotation at the Victoria and Mount Wilson observatories appear to confirm data from variable stars and star clusters indicating that the distance to the center of our own system is of the order of ten kiloparsecs, and suggesting a total diameter of not less than thirty kiloparsecs. But the data on clusters, being incomplete for large distances in low latitude, can indicate only minimum values for the distance to the center and for the total diameter.

We have looked so far without success for other galaxies of as great dimensions as our own. At the distance of three megaparsecs, the largest galaxies in the Virgo group have linear diameters of approximately six kiloparsecs.[1] Hubble and Humason derive a distance of two megaparsecs, with proportionately smaller linear diameters.[2] The largest galaxies in the Centaurus group, according to an earlier estimate of the

distance, appeared to be as large as the Andromeda Nebula,[3] but recent, more definite estimates of the distance and revised measurements of angular diameters give maximum linear diameters of approximately six kiloparsecs.

2. The present note discusses the maximum linear dimensions of the 125 largest systems in the twenty-five groups of galaxies reported in an earlier communication.[4] It is in these physical supersystems that we can most readily and certainly sort out the giants. Isolated galaxies, to be sure, may be systematically different in mass and dimensions from those appearing in clusters; but so far as definite estimates of size have been obtained, the isolated and group galaxies are in the mean closely comparable, and the search for outstanding giants is appropriately turned to the clusters of galaxies because of greater security in estimating the distances and in sorting out the largest.

The examination of the linear diameters indicates that the mean diameter for the five largest in each system is sufficiently alike from group to group to be used as a serviceable indicator of distance, thus supplementing the luminosity and apparent-velocity methods of distance estimation.

3. For each of the twenty-five groups the linear diameters in kiloparsecs of the five largest members are shown in table 1, which gives also the mean of the five, the distance of the group, and the mean density throughout the group expressed in terms of the number of galaxies per cubic centimeter. The distances and densities are taken from the earlier communication on these clusters of galaxies.[4] The angular diameters were measured on long exposure Bruce plates and reduced to linear measures with the aid of the tabulated distances.

Three objects with large angular diameters that fall within the bounds of groups 20, 21 and 23 were rejected as undoubtedly superposed; they are near the apparent boundaries of the groups, and many other objects of their same type and dimensions, appearing on the photographs, are quite clear of the groups themselves. If the three rejects were included as members of the groups in the direction of which they lie, their diameters would be 17, 12 and 22 kiloparsecs, respectively.

4. The tabulated diameters range from 1.0 to 12.7 kiloparsecs; 44 per cent are between 3.0 and 5.0 kiloparsecs. The means range from 1.6 (half weight) to 8.1; and the average deviation from the mean of the means, 4.2 kiloparsecs, is ± 1.0, or less than twenty-five per cent.

The diameters of these 125 galaxies, selected for their large size from 2650 galaxies of which none is less than average size, are far from comparable with the dimensions of our own galactic system. Only a few of these "giants" are more than half as large as the Andromeda Nebula, and none surpasses it. It is unlikely that the diameters in table 1 are underestimated because of systematically too small adopted distances of the

groups; some investigators would argue that the distances are too large. Should future study give smaller distances, it would widen still further the difference between our galactic system and the largest galaxies in these supersystems.

The mean diameter of the 125 largest galaxies is comparable with that of the Large Magellanic Cloud. Among the 2650 systems from which the 125 have been selected, the most frequent diameters lie between 1400

TABLE 1

LINEAR DIAMETERS OF 125 GIANT GALAXIES

GROUP	DIAMETERS IN KILOPARSECS 1	2	3	4	5	MEAN	DISTANCE (MPC)	$\bar{\rho}_1$ (GLX/CC)
1*	4.3	4.0	2.9	2.2	2.2	3.2	8.3	37×10^{-70}
2	8.8	6.2	4.3	3.2	3.2	5.1	11	8.1
3	4.8	3.9	3.5	2.6	2.6	3.5	15	9.4
4	3.8	3.1	2.8	2.8	2.4	3.0	24	1.4
5	9.2	4.1	2.6	2.6	2.3	4.2	10	55
6*	2.6	2.0	1.6	1.6	1.0	1.8	14	68
7	6.3	5.0	3.9	3.7	2.4	4.3	18	7.1
8	6.6	5.8	5.8	5.4	4.4	5.6	25	0.54
9	3.7	3.0	2.6	2.4	1.9	2.7	16	17
10	6.4	3.8	2.9	2.6	2.6	3.7	22	1.2
11	5.5	2.2	2.2	2.2	1.9	2.8	22	2.4
12	4.5	4.5	4.1	3.7	3.2	4.0	28	0.22
13	10.6	8.1	7.2	6.8	5.7	7.7	26	0.66
14	4.0	3.7	3.0	3.0	2.0	3.1	23	1.2
15	7.0	4.3	4.0	3.4	3.4	4.4	23	1 4
16	5.7	5.1	5.1	4.2	3.4	4.7	29	0.69
17	4.6	3.8	3.4	3.0	3.0	3.6	29	0.78
18	4.2	4.0	3.0	2.7	2.7	3.3	26	6.0
19*	2.6	1.8	1.2	1.2	1.0	1.6	14	63
20	12.7	8.4	7.2	6.8	5.5	8.1	29	0.51
21	7.2	4.6	3.4	3.4	3.4	4.4	29	1.5
22	4.5	3.7	3.7	3.2	3.2	3.6	28	2.6
23	5.3	5.3	4.8	2.9	2.9	4.2	33	0.28
24	4.3	3.8	2.9	2.4	2.4	3.2	33	3.3
25	7.7	7.2	5.8	3.8	3.8	5.7	33	1.2

* Half weight because atypical.

and 1800 parsecs, somewhat less than the diameter of the Small Magellanic Cloud. In referring to the diameters of the Magellanic Clouds and the Andromeda Nebula, I do not include the scattered outlying clusters, which are probably satellites of the parent systems, because such objects are not photographed near the faint galaxies, nor are they necessary for the deduction of excessive dimensions of our own galaxy.

5. An examination of table 1 shows that there is a slight tendency for the linear diameters of individual galaxies to be small for high average

density in space. There is a more distinct change of computed mean linear diameter with distance. Both of these tendencies arise almost wholly from the errors in determining distances—an overestimate in distance results in diameters too large and mean densities too low; an underestimate, the reverse, thus forcing the correlation.

It can be shown that a systematic error of this sort in the distances that are derived from luminosity data would occur naturally if in rich groups the individual galaxies are generally more alike in brightness (less spread

TABLE 2

GROUP	d_2 (MPC)	d_r (MPC)	$\bar{\rho}_r$ (GLX/CC)	$\bar{D}_r$ (KPC)
1*	11	9.6	24×10^{-70}	3.7
2	9	10	10.8	4.6
3	18	16.5	7.1	3.8
4	34	29	0.8	3.6
5	10	10	55	4.2
6*	33	23	15	3.0
7	18	18	7.1	4.2
8	19	22	0.8	4.9
9	25	20.5	8.1	3.5
10	25	23.5	1.0	3.9
11	33	27.5	1.2	3.5
12	29	29	0.2	4.1
13	14	20	1.5	6.0
14	26	24	1.1	3.7
15	26	24	1.2	4.3
16	26	27.5	0.8	4.5
17	23	26	1.1	3.9
18	33	29.5	4.1	3.8
19*	37	25.5	10.4	2.9
20	23	26	0.7	6.2
21	23	26	2.1	4.3
22	33	30	2.1	3.9
23	33	33	0.3	4.2
24	31	32	3.6	3.7
25	31	32	1.3	4.9

* Half weight.

in magnitudes) than in the sparsely populated groups; star clusters exhibit such a luminosity peculiarity, as is conspicuously shown by the nearly total absence of supergiant stars from globular clusters and from some of the richest of open clusters.

In view of the probable systematic error in the computed mean diameters because of the effect of the accidental errors in estimating the distances of the groups, a revision with the aid of the data on angular diameters becomes appropriate.

6. Most of the investigators of the subject have concluded that neither

apparent magnitude nor angular diameter is a very useful criterion in the determination of the distances of individual galaxies—the dispersions are too great. But for groups, the objections to the use of magnitudes or diameters are of less force. The data of table 1 show that the average diameter of the five largest in any individual group may be used, especially in connection with the luminosity curves, to compute the approximate distance of the group.

Table 2 contains new values, d_2, of the distances computed on the assumption that the linear diameter is exactly 4.2 kiloparsecs for the mean of the five largest in each supergalaxy. For groups that appear in pairs or triples[4] the means have been taken for ten and fifteen, respectively. Combining these values, with equal weight, with the values, d_1, of the distances derived from the luminosity data (table 1), we have as revised and adopted distances, d_r, the values in the third column of table 2, and in the fourth the corresponding revised value for $\bar{\rho}_r$, the mean density in the group. In the last column are given the mean diameters, $\bar{D}_r$, recomputed on the basis of the adopted distances. The revised values are computed from the relations

$$d_r = {}^1/_2\left(\frac{d_1 + 4.2d_1}{D_1}\right)$$

$$\bar{\rho}_r = \frac{\bar{\rho}_1 d_1}{d_r^3}$$

$$D_r = \frac{(D_1 + 4.2)}{2}$$

The revised mean linear diameters of course show but one-half the spread shown in table 1, the extremes now being 2.9 and 6.2, and the spread in mean space density is also less. The dependence of diameter on distance has entirely disappeared; thus is indicated the appropriateness of the revision in table 2. Although the zero point for the revised distances depends wholly, as before, on the adopted integrated absolute magnitude of the average giant galaxy, I believe that the revised values of table 2 for the distances, diameters and space densities are much better than those given in the earlier paper and in table 1, because now the serious accidental errors have been much decreased.

Summary.—1. Search among the 2650 largest members in twenty-five groups of galaxies fails to show any objects larger than the Andromeda Nebula, and there is nothing at all comparable to our galactic system; if such galaxies exist they must be extremely uncommon.

2. As previously known, the average galaxy is a little less than two kiloparsecs in diameter. The Small Magellanic Cloud and the larger of

the two companions to the Andromeda Nebula are of average size; the smaller Andromeda companion apparently is a dwarf galaxy, and the Large Magellanic Cloud is comparable in dimensions with the mean of the 125 largest in the twenty-five groups.

3. The mean angular diameter of the five largest galaxies in a group (if the group is not too small and scattered) is a fair criterion of distance when the diameters are measured on Bruce plates of long exposure. When current values are accepted for the integrated photographic absolute magnitudes of galaxies,[4] the mean linear diameter of the five largest galaxies in a group may be provisionally taken as 4.2 kiloparsecs.

4. On the basis of the angular-diameter method, the distances have been revised and new values derived for the mean density of matter in space in these twenty-five groups. The revision materially decreases the dispersion in density, but it confirms the earlier result that in the large groups the space density averages about 5×10^{-70} glx/cc, which corresponds probably to a total space density considerably higher than 10^{-28} g/cc.

[1] *H. B.* 873 (1930).
[2] *Mt. Wilson Contr.* 427 (1931).
[3] *H. B.* 874 (1930).
[4] These PROCEEDINGS, **19**, 591 (1933); H. Repr., 92 (1933).

LINKAGE INTERRELATIONS OF THREE GENES FOR REX (SHORT) COAT IN THE RABBIT

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Three races of so-called rex rabbits (in German, Kurzhaarkaninchen) have been produced in recent years. The hair of such animals is abnormally short, soft and plush-like, and their whiskers (vibrissae) are bent or curly. The guard hairs are scarcely, if at all, longer than the other hairs and are often erroneously described as lacking.

Rex rabbits are now being extensively bred by rabbit fanciers. Hope has been entertained by many that the pelts of such animals would be valuable to the furrier, but a decision on this point must probably await the time when such pelts will be available in quantity and carefully graded.

Meantime these new mutations of the rabbit afford material of unusual interest to students of rabbit genetics. For convenience, the three races will be designated rex No. 1, rex No. 2 and rex No. 3. In Germany they are

known as Rex, Deutsch-Kurzhaar and Normannen-Kurzhaar, respectively.[1] All are phenotypically similar, indeed quite indistinguishable, when the residual heredity is the same, but they are all genetically different, so that crosses between any two of them produce rabbits with normal coats.

Rex No. 1 is the one best known to fanciers. It was discovered in France by Abbé Gillet in 1919 as a mutation occurring in a race of ordinary rabbits. Its inheritance, as well as that of the other two rex mutations, is that of a simple Mendelian recessive character.

Several years later, in 1926, a breeder in Hamburg, Germany, discovered a short-haired rabbit in the hutches of another breeder at Lübeck, Germany. He was able promptly to establish from this individual the race Rex No. 2 (Deutsch-Kurzhaar). Still later, in 1927, in another part of France a breeder of Himalayan rabbits, Madame Du Bary, observed that short-haired individuals were occurring among her animals. From these she established a race which we will call Rex No. 3 (Normannen-Kurzhaar).

Breeding stock of rex No. 1 and rex No. 2 was secured by Nachtsheim in Germany, and the initial crosses with ordinary rabbits were made by him. Later, F_1 crossbreds were brought by him or sent to Castle, in America, and crosses were made by Castle, as well as by Nachtsheim, between the two races and with race rex No. 3, which meanwhile had been obtained from Madame Du Bary in France.

In earlier publications both Nachtsheim and Castle have described breeding experiments with rex No. 1, in which it was shown to be a simple recessive character, and to recombine independently of all previously known genes of the rabbit. In other words, it was not linked with any other gene of the rabbit known at that time, and so must be supposed to have its locus in a different chromosome.

Similar linkage studies were undertaken with rex No. 2, and the results in this case also were negative, until a test was made for linkage between the two rex genes, rex No. 1 and rex No. 2. To our great surprise this test was positive, showing that these two rex mutations, due to different and complementary genes, had their loci in the *same* pair of chromosomes. The evidence for this conclusion is as follows.

A cross between rex No. 1 and rex No. 2 produces rabbits entirely normal in appearance, both mutations being completely recessive. When these F_1 normal animals are mated with each other, they produce mixed litters of normal and short-haired individuals, the latter being distinguishable from normals at birth by their curly whiskers.

A total of 391 F_2 young has thus far been recorded, of which 195 were normal and 196 short-haired. If the two genes were independent, located in different chromosomes, we should expect a 9:3:3:1 Mendelian ratio to be approximated in F_2, the last three classes all being short-haired.

The ratio of normals to short-haired would then be 9:7, a decided excess of normals. This does not occur. Instead, the two classes are approximately equal.

If, on the other hand, the two rex genes are completely linked, borne in the same chromosome at identical or closely adjacent loci, then we should expect them to behave like allelomorphs, and to give a 1:2:1 ratio in F_2; and of the three classes the first and last would be short, the middle class being normal. In other words, there should be equal numbers of normal and short-haired, as actually observed. It was not expected, accordingly, that crossovers would be detected among the F_2 individuals. Nevertheless, search was made for them, and, much to our surprise, they were found.

It is impossible, by inspection, to determine whether a particular F_2 short-haired individual is an r_1 or an r_2 individual, though it must be homozygous for one or the other gene, since otherwise its coat would be normal, like that of the F_1 individuals which are heterozygous for both genes.

Accordingly, breeding tests were made of F_2 short-haired individuals by backcrossing them to one or the other, or, if necessary, to both of the pure races. Tests of this sort have been completed for 51 F_2 short-haired individuals. Of these, 18 have proved to be pure r_1r_1 individuals which do not carry the r_2 gene. Their genetic formula is accordingly $r_1r_1R_2R_2$. A like number, 21, have proved to be pure r_2r_2 individuals, whose formula is $R_1R_1r_2r_2$. Five individuals are of the composition $r_1r_1R_2r_2$, i.e., they are homozygous for r_1 and heterozygous for r_2; and seven are of the constitution $R_1r_1r_2r_2$, homozygous for r_2 but heterozygous for r_1. No double recessives have as yet been identified, though their production in F_2 would be theoretically possible, and in F_3 would be expected to occur frequently, if the crossover individuals ($r_1r_1R_2r_2$ and $R_1r_1r_2r_2$) were mated one with another.

The F_1 normal individuals, as already stated, are double heterozygotes, $R_1r_1R_2r_2$. The gametes which they would be expected to produce are $r_1R_2 + R_1r_2$ (non-crossovers) and $R_1R_2 + r_1r_2$ (crossovers). A non-crossover gamete (r_1R_2) uniting with a crossover gamete, r_1r_2, would produce the "crossover class" $r_1r_1R_2r_2$, of which five individuals have been identified, one male and four females. The other sort of non-crossover gamete (R_1r_2) uniting with a crossover gamete, r_1r_2, would produce the "crossover class" $R_1r_1r_2r_2$, of which seven individuals have been identified, four being males and three females.

The question now arises, what percentage of crossover gametes do the F_1 individuals produce? In other words, what is the crossover percentage? Each of the 12 crossover individuals received one crossover gamete and one non-crossover gamete, as already indicated. The 39 other tested

F_2 short-haired individuals received only non-crossover gametes, two each. The total number of non-crossover gametes concerned in the production of the tested F_2 population of 51 individuals is accordingly 12 + 78, or 90; whereas the crossover gametes number 12. This is 11.7 ± 2.1 per cent of the total 102 gametes which entered into the production of the tested F_2 population. The indicated crossover percentage is accordingly between 10 and 12 per cent.

The foregoing calculation takes into consideration only the double recessive type of crossover gamete, r_1r_2. We should expect a second type of crossover gamete, the double dominant, R_1R_2, to be produced in like frequency by the F_1 parents. That it is actually produced has been experimentally demonstrated but not on a sufficient scale to give the results quantitative value. Two of the normal-coated F_2 individuals have been tested by appropriate matings with pure r_1 and pure r_2 races. One of them was found to be, like the F_1 parents, a double heterozygote, $R_1r_1R_2r_2$. It evidently arose from the union of two dissimilar non-crossover gametes, r_1R_2 and R_1r_2. The other individual was heterozygous for r_1 only, its constitution being $R_1r_1R_2R_2$. It could have arisen only from the union of a non-crossover gamete, r_1R_2, with a crossover gamete R_1R_2.

It is accordingly clear that crossing-over does occur between the genetic loci r_1 and r_2, and that they are distant from each other about 10 or 12 units, since about 10 or 12 per cent of crossing-over occurs. It seems surprising that if crossing-over occurs as frequently as this, the F_2 ratio of normal to short-coated young should indicate complete linkage by yielding approximate equality of the two classes. But upon calculating the expectation for F_2 with 10 per cent of crossing-over, it is found to deviate very little from equality, so little, indeed, that random sampling would be likely to conceal it completely. Thus, in an F_2 population of 400 individuals, the expectation is 201 normal to 199 short, if crossover gametes are to non-crossover gametes as 1:9. We have actually observed 195 normals:196 short in a population of 391 individuals. This is very close to the theoretical expectation.

A very different F_2 ratio of normal to short is obtained when either rex_1 or rex_2 individuals are crossed with the rex_3 race. The former cross has produced an F_2 population of 278 individuals, of which 159 are normal and 119 short-haired. A 9:7 ratio, expected if no linkage exists, would be 156.4:121.6, an even smaller excess of normal individuals than is observed. The deviation from expectation is only 2.6, the probable error being 5.6. Accordingly, the agreement with expectation is excellent.

If, however, we test the observed numbers for agreement with a 1:1 ratio, expected if close linkage exists between r_1 and r_3, we find the deviation to be 17.4 and the P. E. 5.58, a very poor fit. In all likelihood, there-

fore, the ratio is 9:7 rather than 1:1, and r_1 and r_3 are independent rather than linked.

The cross between rex_2 and rex_3 has produced an F_2 population of 166 individuals, of which 104 are normal and 62 short-haired, a considerable excess of the former. On a 9:7 basis the expected numbers would be 93.4:72.6, and the excess of normals would be 10 less than observed, the P. E. being 4.3. This is only fair agreement, but the deviation from expectation is in the opposite direction from what would be expected if there were linkage between r_2 and r_3, for in that case normals and shorts should be approximately equal. From this expectation the observed numbers deviate by 21 individuals, the P. E. being 4.3.

Accordingly, in both crosses agreement is better with a 9:7 than with a 1:1 ratio of normal to short-haired, the former ratio indicating complete independence, the latter linkage. Since r_1 and r_2 are linked with each other (lie in the same chromosome), it follows that if one of them is free from r_3 (lies in a different chromosome), then the other must be free also. Accordingly, it is legitimate, for statistical purposes, to combine the two F_2 populations obtained by crossing r_1 and r_2 with r_3. If this is done, we have a total of 444 individuals, of which 263 are normal and 181 short-haired. The expectation on a 9:7 basis is 249.7:194.3, from which the observed numbers deviate by 13.3, which is less than twice the probable error (7.0) and so not significant, since it may well be a consequence of random sampling. It should be noted, also, that the deviation is not in the direction to indicate linkage (toward equality of normal and short) but in an opposite direction toward inequality of the two classes, which is expected if no linkage occurs.

Further evidence that r_1 and r_2 assort freely with reference to r_3 (though not with reference to each other) is found in a few tests which have been made of the constitution of F_2 short-haired animals derived from the crosses $r_1 \times r_3$ and $r_2 \times r_3$. From the former cross, one individual has been found to be $r_1r_1R_3r_3$ and another $R_1r_1r_3r_3$. Each of these must have arisen from the union of a crossover (new combination) with a non-crossover (original combination) of an r_1 with an r_3 allelomorph.

From the $r_2 \times r_3$ cross, three F_2 short-haired individuals have been fully tested. One is of the constitution $r_2r_2R_3r_3$, another is $R_2r_2r_3r_3$ and a third is $r_2r_2r_3r_3$. Into the production of the first two there entered, in each case, one *new combination* and one *original combination* gamete. Into the production of the third, only *new combination* gametes entered.

It appears, accordingly, that *new combination* gametes are fully as frequent in occurrence as *original combination* gametes. We have evidence that in the production of these five tested F_2 individuals 6 of the former sort and 4 of the latter sort were concerned. If there were linkage between r_3 and the other rex genes, we should expect *new combinations*

to be less frequent in occurrence than *original combinations*, but they are not.

The somatic character of the individual homozygous for two rex genes, r_2 and r_3, is of interest. Each of these genes, by itself, is able to produce the same short-haired type. Together they produce apparently no greater effect than each would produce by itself.

Summary.—Three races of short-haired (rex) rabbits have originated, each by a recessive mutation in a different gene. The genes responsible for two of these rex mutations (r_1 and r_2) are borne in the same chromosome, since they are linked with about 10 or 12 per cent of crossing-over between them. The gene for the third rex mutation (r_3) lies in a different chromosome, since it recombines freely with both r_1 and r_2.

[1] In Nachtsheim's laboratory the genetic symbols used are:

rex = Franz. Rex = r_1
dek = Deutsch-Kurzhaar = r_2
nok = Normannen-Kurzhaar = r_3.

THE GENE THEORY IN RELATION TO BLENDING INHERITANCE

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Before the rediscovery of Mendel's law, Francis Galton had recognized two sharply distinct classes of inherited characters, those which are alternative and those which blend. He found examples of these in human eye color and human stature, respectively. When the parents differ in eye color, the children are like one or the other parent, the inheritance being characterized as alternative; but when the parents differ in stature, the children are of intermediate stature, a blending effect.

With the rediscovery of Mendel's law, it was recognized that an adequate explanation had been found for the inheritance of alternative characters, which were then commonly called unit-characters. But this terminology later fell into disrepute because it was found that literal-minded individuals conceived of the character itself as being inherited in the gametes rather than a single something which was necessary for the production of the character. So the term *gene* was introduced to designate the indispensable something in the germ cell necessary for the development of the character in the individual.

By a brilliant series of discoveries, Morgan and his pupils proved that the genes are located in the chromosomes of the germ cells where they

have a definite arrangement in linear order. The problem of heredity was thus *solved*, as Morgan stated in one of his publications, but only as regards what Galton termed alternative characters. As regards the inheritance of blending characters, a problem still remained unsolved. To its solution I had devoted considerable attention, even before the establishment of the chromosome theory. I found that body size in rabbits behaves as a blending character, regularly producing intermediates, like human stature, according to Galton. This is the unquestionable result in a great number of quantitatively varying characters in animals and plants; and, in general, it is the more fundamental and essential characters of organisms which blend in their inheritance, whereas the more superficial and inconsequential characters mendelize. Accordingly, an understanding of the nature of blending inheritance is of prime importance.

A hypothesis that blending characters, as well as alternative characters, are determined by chromosomal genes gradually gained acceptance, based primarily on the experimental work of Nilsson-Ehle, East and Emerson, and many others. It rests on the following assumptions: (1) that genes located in many different chromosomes affect such a quantitatively varying character as body size; (2) that they are inherited independently, but (3) that their action is cumulative or additive. Thus, genes *ABCD* produce a large animal. *BCD* produce a smaller one, *CD* a still smaller, and so on. This so-called multiple factor explanation of blending inheritance is ingenious and plausible and, at present, generally accepted among students of genetics, but it lacks demonstrative proof. For many years I have sought critical evidence for or against it in crosses between large-bodied and small-bodied races of rabbits, but thus far in vain. I have only negative evidence of the existence in the chromosomes of genes influencing body size. Nevertheless, I have little doubt that they exist. We know that chromosomal genes influence the size, both absolute and relative, of particular parts of the body in animals, and of parts such as leaves, stems and fruits in plants. Thus there is in one race of mice a mutated gene which produces an abnormally short ear. It is borne in the same chromosome as a gene which produces a blue rather than a black coat. There is in mice another mutated gene which produces a generally dwarfed body through reduced activity of the anterior pituitary gland. Both of these genes are inherited in regular alternative mendelian fashion. These cases prove only that body size may be *influenced* by chromosomal genes; they do not prove that the size of an organism is regulated *exclusively* by the action of genes, which is the assumption underlying the gene theory. There has to be an organism before influences can change it in any way. The organism is, I believe, more than a bundle of influences, just as a man is something more than a bundle of endocrine

glands, even though these may profoundly influence his personality and behavior.

The gene theory ignores certain of the findings of embryology which run counter to it. Thus Conklin, Spemann and others have shown that an organization is present in the cytoplasm of the egg at the beginning of development which determines, in a general way, what organs and tissues shall arise from each part of it. The determination is not at first complete and final but becomes progressively more so as development proceeds. In the amphibian egg, at the outset of development, a center of organization, the so-called organizer, determines what organ system shall arise from each of the adjacent parts of the egg. If the organizer is removed from its usual position to another part of the same egg or of a different egg, even to an egg of another species, it still has the power to organize adjacent structures into an amphibian embryo. This is not gene action, for all cells of the embryo have by hypothesis the same gene constitution, irrespective of whether they lie within or without the organizer territory. It is action by the cytoplasm directed by a cytoplasmically differentiated organizer.

But the gene theory holds that the cytoplasmic organization was itself impressed on the cytoplasm by the genes within its nucleus. This is an unproved assumption, for only amphibian cytoplasm containing amphibian genes can produce an amphibian embryo. Both are indispensable, and neither can function without the other. There is no reason to think that historically either has ever existed except in association with the other.[1] Genes may indeed change by mutation, singly or in groups, and thus alter the character of the organism. The cytoplasm also undergoes slow changes, since it has a specifically different character in species or genera which presumably had a common ancestry. But there is no more reason for supposing that cytoplasmic changes are initiated in the gene complex than for the contrary assumption that gene changes originate in the cytoplasm. The studies of Wettstein on plasmatic inheritance in mosses indicate that foreign chromosomes have no modifying action on the cytoplasm even when introduced into it generation after generation.

There is an undoubted mechanism for the inheritance of mendelizing characters in the chromosomes. From the behavior of the chromosomes in fertilization and in maturation of the gametes, alternative inheritance is the only possibility when maternal and paternal gametes differ in their gene content. But blending inheritance is not primarily gene determined unless, as commonly assumed, it is indirect and *masked* gene inheritance, an assumption as yet unproved.

The cytoplasm offers a mechanism suitable for the inheritance of blending characters. The general organization characteristic of the species is already present in the cytoplasm of the egg before fertilization. The

sperm introduces in its chromosomes potential modifiers of this organization which act as accelerators or inhibitors of particular processes in the developing organism, but do not affect the general features of that organization. A similar set of modifiers was already present in the egg pronucleus. The fertilized egg will develop under the joint influence of the two sets of modifiers. If those two sets are exactly alike, gene for gene, then the organism will be completely homozygous and as similar to the mother as asexually produced offspring would be, for it would have cytoplasm derived presumably without change from the mother (or from both parents jointly), and its gene equipment would be identical with that of the mother, provided the mother was also homozygous.

If, however, the two sets of genes are different in any respect, then the modifiers of the organism will be different from those found in the mother. Even though the cytoplasm is identical in character in mother and offspring, the difference in genes acting upon it in the two cases will produce a different result in each.

If the sperm introduces cytoplasm as well as chromosomes into the egg at fertilization (a matter at present uncertain), then there must be a blending of plasms which would account for the blending character in inheritance of general features of organization, such as body size.

Blending characters are the more important and essential ones, those which characterize species, genera and families. Alternative characters (gene determined) as a rule characterize individuals and are more special in nature. There are indeed cases in which Linnean species have been shown to differ by one or more mendelizing characters, but these are rather accidental differences than otherwise. Species crosses usually result in blending inheritance of the general characters of the organism, even when a mendelizing character difference is also involved. The former are the usual and essential differences, the latter are rare and relatively non-essential.

In favor of the hypothesis that blending characters are gene determined is often cited the fact that, as a rule, F_2 is more variable (fluctuates more widely) than F_1. This is assumed to be a consequence of the recombination of multiple factor genes, which undoubtedly act as modifiers of the character in question. But these modifiers may be less important than the organization modified, which finds its expression in the cytoplasm itself. There is reason to believe that the general organization is primarily plasmatic and is derived either from the mother alone or from both parents, if the sperm contains cytoplasm.

On this view racial differences in body size might be due in part to plasmatic differences and in part to gene differences but the increased variability of F_2 as compared with F_1 would be due to gene differences

alone, since blending of plasmas would presumably be as complete in F_2 as in F_1.

In so far as size inheritance is plasmatic, there should be a difference between reciprocal crosses of large-bodied and small-bodied races, since it is reasonable to suppose that the egg plasm is more influential, as it is more abundant, than any cytoplasmic contribution of the sperm. Consequently, we should expect the mother to be more influential than the father in determining the body size of the offspring. In reciprocal crosses between horse and ass this is reported to be true, but the correctness of the report has been challenged, and it must be regarded, for the present, as lacking critical verification. I have reported no difference as being found between reciprocal crosses of large race and small race rabbits, but the number of observations on offspring of small race mothers was small, and no great attention, at the time, was given to the matter which merits renewed investigation.

Summary.—The chromosomes are the undoubted mechanism for the transmission of characters which follow Mendel's law in inheritance. But the more general features of organization are not so inherited. They follow a law described by Galton as blending inheritance. The cytoplasm of the egg-cell affords a suitable mechanism for the transmission of blending characters, although it must be recognized that genes borne in chromosomes may modify such characters. Evidence is needed as to whether the sperm as well as the egg functions in plasmatic inheritance.

[1] In plants such as mosses, where the male gamete resembles an animal spermatozoon in structure, the inheritance has been shown by Wettstein in certain wide species crosses to be such that only maternal chromosomes in maternal cytoplasm are viable, paternal chromosomes in maternal cytoplasm being not viable in the haploid state, though they persist and exert a somatic influence in the diploid generation.

THE EFFECT OF X-RAY DOSAGE ON STERILITY AND NUMBER OF LETHALS IN DROSOPHILA MELANOGASTER

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Abundant evidence is available to show that the effect of x-rays on sterility and on the production of lethals is proportional to the dosage applied. Scattered data dealing with this subject have been summarized in papers appearing recently (Timofeeff-Ressovsky, 1931; Oliver, 1934). From the material published it is evident that standards used in measuring

TABLE 1

AVERAGE NUMBER OF OFFSPRING PER MALES TREATED WITH VARIOUS X-RAY DOSAGES AND MATED WITH ATTACHED-X FEMALES

TREATMENT r-UNITS	MALES TREATED	MALES FERTILE	AVERAGE NUMBER OF OFFSPRING PER TREATED MALE ♀	♂	TOTAL	FERTILE MALE ♀	♂	TOTAL
Control	25	24	40.32 ± 2.33	67.20 ± 3.55	107.52 ± 5.68	42.00 ± 2.13	70.00 ± 3.15	112.00 ± 5.04
608	25	24	30.12 ± 1.86	54.12 ± 3.12	84.24 ± 4.81	31.38 ± 1.74	56.38 ± 2.85	87.75 ± 4.39
1215	22	22	30.18 ± 1.40	47.77 ± 2.11	77.95 ± 3.17	30.18 ± 1.40	47.77 ± 2.11	77.95 ± 3.17
1823	25	23	13.96 ± 1.06	21.68 ± 1.80	35.64 ± 2.61	15.17 ± 0.98	23.57 ± 1.82	38.74 ± 2.60
2127	25	21	8.04 ± 0.68	13.48 ± 1.33	21.52 ± 1.95	9.57 ± 0.58	16.05 ± 1.27	25.62 ± 1.76
2430	35	29	9.51 ± 0.84	13.23 ± 1.04	22.74 ± 1.80	11.48 ± 0.83	15.97 ± 0.95	27.45 ± 1.64
2734	50	46	9.74 ± 0.61	16.44 ± 0.98	26.18 ± 1.52	10.59 ± 0.59	17.87 ± 0.94	28.46 ± 1.45
3038	104	72	3.99 ± 0.30	6.70 ± 0.50	10.69 ± 0.77	5.76 ± 0.36	9.68 ± 0.58	15.44 ± 0.87

dosage applied were not uniform since the discrepancy between various results is greater than the experimental error should permit.

Preliminary to a long series of experiments in which x-rays were to be used it was thought advisable to determine the most efficient treatment for our purposes. Two sets of experiments, therefore, were conducted with treated males: one to determine the relation between the x-ray dosage and the sterility, and the other to determine the relation between the dosage and the number of lethals induced.

In these experiments the dosage was measured with the instrument which has been just calibrated by the Bureau of Standards, the time elapse between the calibration and the measurement never exceeding six weeks. It is reasonable to suppose that the dosage measurements in these experiments approximated the standard units as close as any data previously published and therefore it seems justifiable to publish the results.

TABLE 2

NUMBER OF LETHALS INDUCED IN MALES BY DIFFERENT X-RAY DOSAGES

TREATMENT *r*-UNITS	NUMBER FERTILE CULTURES	NUMBER LETHALS	PER CENT LETHALS
Control	396	2	0.50 ± 0.24
304	479	8	1.67 ± 0.39
637	482	25	5.19 ± 0.68
1215	314	26	8.28 ± 1.05
2186	401	46	11.47 ± 1.07
3038	118	17	14.41 ± 2.18

Methods.—The x-ray radiation was applied by a Universal Type Coolidge tube, with a tungsten target, manufactured by the Victor X-Ray Corporation. The voltage and the milliamperage adjustments were not changed during the experiments. The current used was about 7 milliamperes at 95 kilovolts. The rays were filtered through a half-millimeter aluminum filter and had a wave-length of about 0.72 Ångström.

The x-ray dosage was measured by a Fricke-Glaser dosimeter, manufactured by the Victoreen Instrument Company. In making measurements, a $^1/_{20}$ cubic centimeter chamber was used by which the total dose applied can be measured. The gelatin capsules with flies were placed adjacent to the dosimeter chamber and to correct for the absorption of the gelatin wall of the capsule a capsule cover was slipped over the chamber also. As the calibration of the Bureau of Standards indicates, 1000 on the scale of the dosimeter is equivalent to 607.6 *r*-units.

In both experiments the three days' old wild-type males of Florida stock were used. The food was the standard corn-meal-agar medium. In the male sterility experiment, each treated male was mated with two attached-*X* yellow females. On the seventh day after the matings were

made, the parent flies were removed from the culture bottles. The flies of low dosage treatments, including the 2127 *r*-unit treatment, were raised in one-half pint culture bottles, the flies treated with 2430 and 2734 dosages were raised in one-quarter pint culture bottles and the flies treated with 3038 *r*-units were raised in 2.5 × 10 centimeter shell vials.

In the experiment to determine the frequency of lethals the "Cl B" (Muller, 1927) method was used. The treated males were discarded three days after the treatment and the females were kept in culture bottles four days longer than the males. All F_2 cultures which contained 25 or

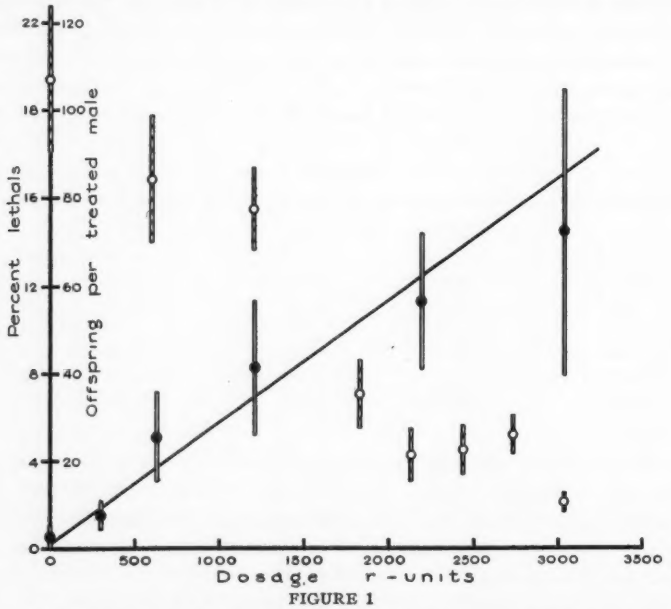

FIGURE 1

Percentages of sex-linked lethals (solid lines) and average numbers of offspring per treated male (broken lines) after a treatment of males with different x-ray dosages.

more flies and had no males were considered as carrying a lethal induced by treatment.

Results.—A summary of the results is given in tables 1 and 2 and a graphic presentation in figure 1. It is evident from the results that both the induced sterility of the males and the frequency of induced lethals is approximately proportional to the dosage applied.

Probable errors for figures given in table 1 were calculated by $\pm 0.6745 \frac{\sigma}{\sqrt{n}}$ formula and probable errors for figures of table 2 were calculated by

$\pm 0.6745 \sqrt{\frac{p \times q}{n}}$ formula.

Discussion.—Under more or less ideal conditions, viz., when each treated wild-type male is mated individually to two females, the 3038 *r*-unit treatment produced on the average 6.7 male and 4 female offspring. In large scale experiments, however, it would not be economical to breed each male separately and therefore a lower productivity could be expected.

In the majority of the large scale experiments which we conducted so far three females were mated with five treated males and the parents were kept in quarter-pint culture milk bottles on corn-meal-agar culture media. In one series of such experiments, where treated white males were mated to attached-*X* yellow females, in 2521 cultures, 33,069 male offspring were obtained after 3038 *r*-unit treatment; and in 963 cultures, 21,136 males were obtained after 2430 *r*-unit treatment. The average number of male offspring per treated male was 2.6 for the 3038 *r*-unit treatment and 4.4 for the 2430 *r*-unit treatment. The productivity of treated males bred under these conditions was so low in the 3038 *r*-unit treatment that a treatment of about 2500 *r*-units was adapted for our experiments.

In another set of experiments where females were examined for variants, 906 cultures for each of which three females and five x-rayed (2430 *r*-units) bar or yellow males were used produced 23,682 females or 5.2 female offspring per one treated male. In that experiment also the males from the 2430 *r*-unit treatment yielded a fair number of female offspring.

After the publication of Gowen and Gay's paper (Gowen and Gay, 1933), in which the effect of treatment with different dosages up to 35,680 *r*-units was discussed, an experiment was conducted in which males were subjected to a treatment with high dosages. Thirty males were used for every treatment. Four days after a treatment with 6070 *r*-units, 23 males were alive and they produced six flies; from a treatment with 9114 *r*-units 20 males were alive at the end of four days but they gave no offspring; and four days after a treatment with 12,156 *r*-units all males were dead and they also produced no offspring. These results make it probable that the dosage used by Gowen and Gay were lower than indicated in their publication. If their curve on the relation between the x-ray dose and the rate of lethal mutation is compared with the one given here, the agreement between the data would be good if the figures for the Gowen and Gay's dosages were reduced to about one-fourth.

In the experiment on the relation between the rate of primary non-disjunctions and the x-ray dosage in *Drosophila virilis* (Demerec and Farrow, 1930) the measurements were made with the same dosimeter chamber as the measurements reported in this paper but the scale on the instrument was read as 1000 instead of 608, as the Bureau of Standards cali-

bration indicates. The figures for dosage reported in that paper, therefore, should be multiplied by 0.608.

Summary.—The data are given for the relation of the rate of lethal mutations and the dosage and the fertility of males and the dosage. The x-ray dosage measurements were made with the $^1/_{20}$ cubic centimeter chamber of the Fricke-Glaser dosimeter which has just been calibrated by the Bureau of Standards.

Experience indicates that for large scale experiments where treated males are mated in mass matings the 2500 *r*-unit exposure is the most advantageous one to use.

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Timofeeff-Ressovsky, N.W., "Die bisherigen Ergebnisse der Strahlengeneti," *Ergebnisse med. Strahlenforsch.,* **5,** 129–228 (1931).

THE SPECIFIC FORMULA OF HEREDITY

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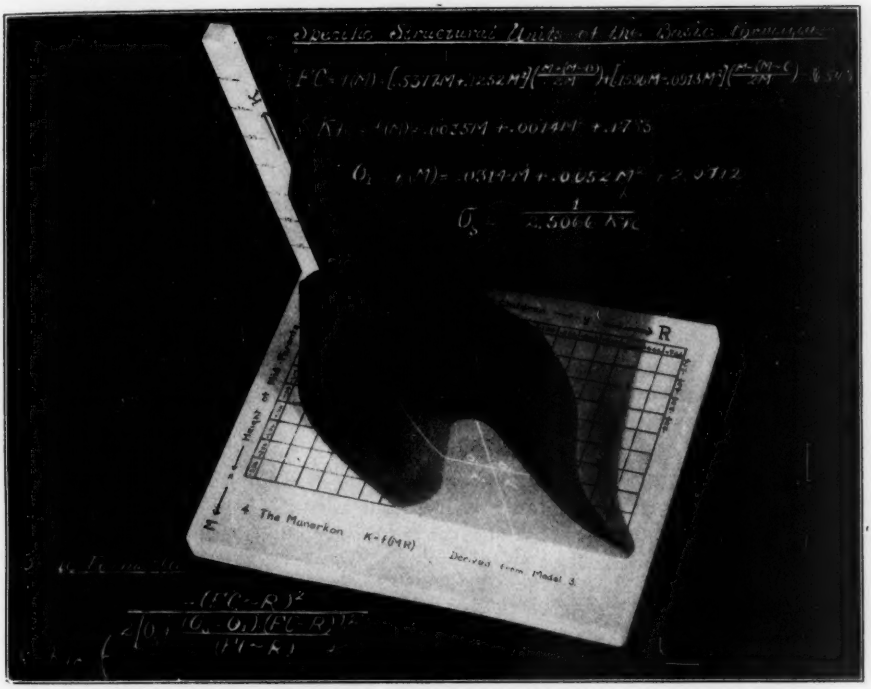
Communicated November 10, 1933

It is demonstrated that, for any measured quality which tends to run-in-the-family, the Specific Formula can be found in conformity with the General Formula, provided that a sufficient number of individual data are available.

M can be any Prediction-basis, provided that it is in reality correlated in some measure with the Thing-predicted. While *M* is continuous in value, *R*, or the Thing-predicted, is also continuous; but for any one selected value of *R* there is always included a range—plus and minus—of definite value. This range may be as small or as great as we may choose. Generally the smaller the *R*-range, and the more numerous and accurate the data, the more exactly will the three-dimension data-graph and the mathematical model of the Specific Formula resemble each other.

In applying the Pattern Formula to a specific case, it is, of course, necessary to find a definite value for each of the eleven constants of the Pattern Formula. Such specific values give individuality to each Specific



THE MATHEMATICAL MODEL FOR THE SPECIFIC FORMULA OF HEREDITY,

by which Nature governs the transmission of stature in that section of the British people sampled by Sir Francis Galton in his classical stature-study, 1884 - 1889.

General Pattern Formula $K = f(M,R)$

K is the probability that a pre-indicated or random-selected offspring, with a given M or prediction basis, will fall within the selected R or class-range of offspring.

In the present specific formula M = Height of Mid-parent in inches - 68.54

Mid-parent = (Height father + Height mother · 1.08) ÷ 2

R = Offspring-stature in inches - 68.54 ± .5 inch.

This locates the offspring-prediction-range of one inch.

Given any selected M and R, one can find the corresponding K, either by substituting the selected values of M and R in the above formula, or by locating the manerkon-surface-point common to the selected M and R, then by reading the coördinate value of K on the "post"

"M" is for Manton, meaning prophecy; "R" for Ergon, meaning the actual thing; and "K" for Eikon, meaning likelihood. Hence is derived "Manerkon" for the name of the three-dimension model, and "Manerkonic" for the present type of prediction-mathematics and of genetic analysis.

In the specific formula for the inheritance of any quality whatsoever - structural or functional, in plants, animals or man - the mathematical formula and model will always be of the general type above shown.

See "General Formula of Heredity" by Harry H. Laughlin, Proceedings of the National Academy of Sciences, Vol. 19, No. 8, pp. 789-801, August 1933.

Formula and to its mathematical model. In each Specific Formula these values are reflected by individuality in the three structural units of the Manerkon: (1) In diagonalness and bend of the center base line $FC = f(M)$; (2) in successive maximum heights always directly above FC, $K_{fc} = f(M)$; and (3) in steepness of the surface, $\sigma_1 = f(M)$.

Each transverse section of the Manerkon will always show a probability curve, generally skewed, and, for the same model the total area of every such cross-section will always equal 1.000 or the probability of certainty, when measured in strips one R-unit wide.

The shape of the mathematical model reflects the genetic constitution of the subject-quality in the population which the model describes—the taller and steeper a section of the Manerkon, the more purely does that particular stretch of the population breed—the more homozygous the particular parents. On the other hand, where the Manerkonic section is relatively low, it means that for such M-region there is a wide range of offspring for each given prediction-value, and that more interacting genes, or more heterozygosity, are involved. In any case, the three coördinate scales being constant, the taller, thinner and more diagonal the Manerkon the more accurate the prediction—the more correctly is Nature's behavior pictured mathematically.

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*A CYTOGENETICAL STUDY OF A RECIPROCAL TRANSLOCATION IN ZEA**

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Communicated October 20, 1933

The ten chromosomes comprising the monoploid set in Zea have been shown by McClintock[1] to be morphologically distinguishable from one another. Each of the ten known linkage groups in maize has been associated with a specific member of the monoploid set. For example, the genes of the lg_1-gl_2-b-v_4 linkage group are situated in the second longest chromosome (No. 2) while the genes of the v_2-pr-bm_1 linkage group are situated in the fifth longest chromosome (No. 5). This association of linkage groups with specific members of the chromosome set has been demonstrated chiefly by McClintock (largely unpublished) through her study of the different primary trisomes and the work of Burnham[2] and Anderson (unpublished) on reciprocal translocations in Zea.

The morphology of the second longest chromosome as determined by measurements at pachytene is as follows:**

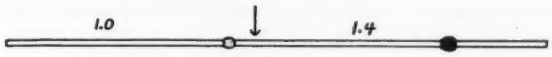

DIAGRAM 1

Schematic representation of morphology of No. 2 chromosome. The position of insertion region shown by stippled region. The location of prominent knob is in the long arm. The relative ratio of the two arms is 1.0:1.4. The approximate point of breakage is shown by the vertical arrow.

and the morphology of the fifth longest chromosome is:

DIAGRAM 2

Schematic representation of morphology of No. 5 chromosome. Insertion region shown by stippled area. Knob is in long arm. Ratio of arms is 1.0:1.1. The approximate point of breakage is shown by the arrow.

A reciprocal translocation between the No. 2 and No. 5 chromosomes was found in untreated material. This reciprocal translocation led to approximately 50 per cent abortion of both ovules and pollen and was designated semi-sterile-4 since it was the fourth case of a similar nature found in Zea. The abortion of one-half the ovules and pollen results from the fact that the distribution of the four chromosomes in the ring of 4 is such that one-half of the gametes are deficient for a part of either the No. 2 or the No. 5 chromosome. In a previous paper[3] the writer reported some linkage values obtained in a study of this translocation. The present paper will present the completed cyto-genetic study of this translocation. No attempt will be made here to review the literature dealing with translocations. This will doubtless be done when the comprehensive studies of Anderson and Burnham on translocations in Zea are published.

Cytological Observations.—Cytological studies at pachytene showed the points at which the translocation occurred in the two chromosomes. This is illustrated in diagrams 1 and 2 where the arrows mark the approximate points at which the breaks occurred in the two chromosomes.

DIAGRAM 3

One of the two new chromosomes formed as result of interchange between No. 2 and No. 5 at the points indicated by the arrows in diagrams 1 and 2. The insertion region is from No. 2.

DIAGRAM 4

The other new chromosome formed by the interchange. The insertion region belongs to No. 5. This new chromosome is marked by a large knob in both arms.

The two newly constituted chromosomes resulting from the translocation are shown in diagrams 3 and 4. Over twenty clear figures of the translocation complex were found at pachytene in which it was possible to identify the insertion regions, the center of the cross, i.e., the point at which the exchange of chromosome homologies occurred and to follow the four arms of the quadripartite figure. As the diagrams show the point of translocation in No. 2 occurred in the long arm but near the insertion region while the point of translocation in No. 5 was in the short arm but also close to the insertion region. The distance from the point of translocation to the insertion region in No. 2 is equal, or nearly so, to the distance from the point of translocation to the insertion in No. 5. Burnham[4] and McClintock[5] have shown that "sliding," which results in different positions for the point of exchange of chromosome partners in the quadripartite figure, occurred in the translocations they studied. This "sliding" or pairing of non-homologous regions makes it difficult to determine the exact positions (cytologically) of the points at which the translocation took place. This same phenomenon of "sliding" was found in the writer's translocation and often resulted in the point of exchange of partners being adjacent to the two insertion regions. The fact that the points of translocation occurred so near the insertion regions and the fact that a marked tendency was found for the center of the cross to lie close to the insertion regions, due to "sliding" or the pairing of non-homologous regions, made the cytological determination of the nature of this translocation rather difficult. The writer is greatly indebted to Dr. McClintock for her invaluable help in interpreting the cytological figures. A camera lucida drawing of the translocation complex at pachytene is shown in diagram 5.

With the occasional exception of a chain of 4 the only configuration observed at diakinesis was a ring of 4. The arrangement on the metaphase plate was of two types: (1) an open ring, or (2) a zigzag arrangement. In the first type the adjacent chromosomes pass to the same pole to give inviable combinations while in the second type of arrangement alternate chromosomes pass to the same pole to give viable combinations of two classes: (1) the two normal chromosomes and (2) the two interchanged chromosomes. The two types of arrangement on the metaphase plate probably occurred with equal frequencies but no extended counts were made.

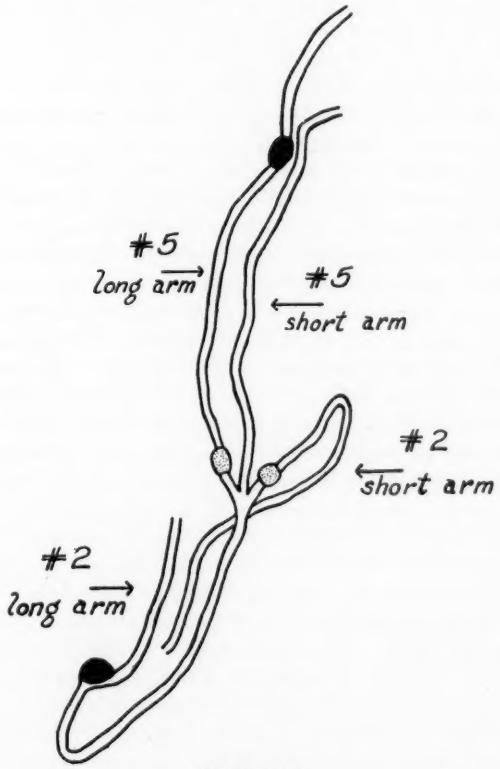

DIAGRAM 5

A camera lucida drawing of the translocation complex at pachytene. The large knob found on the long arm of No. 2 is in a heterozygous condition while the knob on No. 5 is homozygous. The two insertion regions are shown by the stippled areas. Each line represents a single chromosome.

Genetic Studies.—Listed below are the percentages of recombination found in backcross progenies for several genes in the No. 2 linkage group with the point of interchange or translocation:

lg_1-*tr* = 44.2% recombination based on a total of 2988 plants
gl_2-*tr* = 31.3% recombination based on a total of 1870 plants
b-*tr* = 22.0% recombination based on a total of 4399 plants
ts_1-*tr* = 4.5% recombination based on a total of 767 plants
v_4-*tr* = 7.3% recombination based on a total of 2222 plants

Detailed data will be presented only for the backcross of individuals heterozygous for *lg*$_1$-*gl*$_2$-*b*-*v*$_4$ and the point of translocation. These data (table 1) show the point of translocation to fall between *b* and *v*$_4$. Other backcross progenies involving *lg*$_1$-*b* and *ts*$_1$ place the point of interchange to the right of *ts*$_1$ so it is possible to state the following order: *lg*$_1$-*gl*$_2$-*b*-*ts*$_1$-*tr*-*v*$_4$. McClintock[6,7] has demonstrated that *lg*$_1$ is near the left end of the short arm and that *b* is situated in the middle portion of the short arm of No. 2. Her studies would also suggest that the locus of *v*$_4$ lies to the right of the large knob in the long arm of No. 2 but owing to an uncertain classification for the *v*$_4$ character this location may be considered doubtful for the present. Since the break in the long arm of No. 2 occurred at a point close to the insertion region it is likely that *ts*$_1$ is in the short arm of No. 2. The *v*$_4$ gene is then the only factor of the No. 2 linkage group of 9 genes whose location is definitely in the long arm. On the basis of the available crossover data the *lg*$_1$ *gl*$_2$ *d*$_5$ *b* *sk* and *ts*$_1$ genes are almost certainly in the short arm while the location of *fl* and *ba*$_2$ is uncertain.

Since the point of translocation in chromosome 2 occurred very close to the insertion region but in the long arm the amount of crossing-over between *b* and the translocation should be approximately equal to the amount of crossing-over between *b* and the insertion region. McClintock has shown that *lg*$_1$ lies near the distal end of the short arm and that *b* is situated in the midportion of the short arm of chromosome 2. That is, the physical distance from *lg*$_1$ to *b* is of the same order as the distance from *b* to the insertion region. The data in table 1 give the amount of crossing-over between *lg*$_1$ and *b* as 35.9 per cent, while the amount of crossing-over between *b* and the translocation is 16.2 per cent. But the amount of crossing-over between *b* and the insertion region must be nearly equal to, if anything less than, the amount of crossing-over between *b* and the translocation. These facts seem to indicate that there is roughly twice as much crossing-over in the distal half of the short arm as there is in the proximal half. This conclusion would agree with the work of Dobzhansky on the genetic and cytological maps of the two autosomes of Drosophila melanogaster, who showed that the amount of crossing-over per unit of physical length was less near the insertion region than in more distal portions. However, a more precise determination of the locus of *b* is needed before a regional difference in crossing-over can be said to be completely established for chromosome 2 of Zea since a shift of the locus of *b* to the right of the midpoint of the short arm would vitiate the regional difference.

The recombination values between the point of interchange and four genes in the No. 5 chromosome were obtained. These values are as follows:

v$_2$-*tr* = 50.8% recombination based on a total of 1704 plants
pr-*tr* = 19.3% recombination based on a total of 2424 plants

bt_1-*tr* = 1.5% recombination based on a total of 1511 plants
bm_1-*tr* = 1.5% recombination based on a total of 1365 plants

Three point backcross data indicate that the order is v_2-*pr*-bm_1-bt_1-*tr* although the relative order of bm_1 and bt_1 is uncertain and may be the reverse of that listed. The location of the point of interchange to the right of bm_1 and bt_1 is based in part on the frequency of double and triple crossovers and it is possible that both bm_1 and bt_1 might be to the right of the point of breakage. However, McClintock[7] (and unpublished) in her study of ring fragments which included the bm_1 locus believes the location of bm_1 to be in the short arm and very close to the insertion region. These facts strongly suggest that bm_1 is in the short arm of No. 5 and somewhere in the region between the insertion and the point of interchange. Since bt_1 gave the same amount of crossing-over with the break or point of interchange as did bm_1 it can be argued, but less convincingly, that bt_1 is also in the same region of the short arm as bm_1. Strong evidence, however, was obtained through a study of $2n + 1$ individuals which arose through a 3 to 1 distribution of the four members of the ring that the order is v_2-*pr*-*bt*-*tr*.

The locus of bm_1 has been shown to be near the insertion region in the short arm of No. 5. Since there is no reduction in crossing-over between *pr* and bm_1 in individuals heterozygous for the translocation it can be argued that the *pr* and v_2 loci are situated in the long arm of No. 5. Irrespective, however, of whether or not *pr* and v_2 are in the long or short arm of No. 5, the data show that v_2 must occupy a more distal position than *pr* and that they are in the same arm. The results of the writer[8] on chromatid crossing-over in No. 5 trisomes would also place v_2 more distant from the insertion than *pr*. The location of bm_1 near the insertion is also in agreement with unpublished data on chromatid crossing-over in which the frequency of exceptional bm_1 trisomes is very low.

Pseudo-Linkage of pr and bt with b.—Since viable gametes are produced only when alternate chromosomes in the ring of 4 pass to the same pole it would be expected that a factor in the No. 2 chromosome which was situated near the point of interchange would show linkage with a gene in the No. 5 chromosome whose position was also near the point of interchange in that chromosome. The intensity of this "pseudo-linkage" would depend upon the amounts of crossing-over between the two genes and the points of translocation. The factor *b* shows about 22% of recombination with the point of interchange in No. 2 while *pr* and bt_1 gave 19 and 1.5 per cent of recombination, respectively, with the point of interchange in No. 5. Therefore, cultures segregating for *pr* bt_1 *b* and the translocation should show a rather loose linkage between *pr* and *b* and a stronger linkage between *bt* and *b*. In table 2 are listed such data from

backcross progenies which were segregating for the three genes and the translocation.

An inspection of table 2 discloses the striking differences in the amounts of crossing-over in mega- and microsporocytes. Unfortunately most of

TABLE 1

DATA FROM CROSS

REGIONS	$\frac{lg_1\ gl_2\ b\ +\ v_4}{lg_1\ gl_2\ b\ +\ v_4}$ ♀ × $\frac{lg_1\ gl_2\ b\ +\ v_4}{+\ +\ +\ tr\ +}$ ♂	
0	$lg_1\ gl_2\ b\ +\ v_4$ = 370	
0	$+\ +\ +\ tr\ +$ = 409	
1	$lg_1\ +\ +\ tr\ +$ = 103	
1	$+\ gl_2\ b\ +\ v_4$ = 99	
2	$lg_1\ gl_2\ +\ tr\ +$ = 111	
2	$+\ +\ b\ +\ v_4$ = 116	
3	$lg_1\ gl_2\ b\ tr\ +$ = 96	
3	$+\ +\ +\ +\ v_4$ = 97	
4	$lg_1\ gl_2\ b\ +\ +$ = 31	
4	$+\ +\ +\ tr\ v_4$ = 21	
1–2	$lg_1\ +\ b\ +\ v_4$ = 6	Region (1) = 17.3% crossing-over
1–2	$+\ gl_2\ +\ tr\ +$ = 10	Region (2) = 18.6% crossing-over
1–3	$lg_1\ +\ +\ +\ v_4$ = 9	Region (3) = 16.2% crossing-over
1–3	$+\ gl_2\ b\ tr\ +$ = 4	Region (4) = 8.0% crossing-over
1–4	$lg_1\ +\ +\ tr\ v_4$ = 8	Coincidence values:
1–4	$+\ gl_2\ b\ +\ +$ = 18	Regions 1–2 = 0.63
2–3	$lg_1\ gl_2\ +\ +\ v_4$ = 9	Regions 1–3 = 0.32
2–3	$+\ +\ b\ tr\ +$ = 6	Regions 1–4 = 1.18
2–4	$lg_1\ gl_2\ +\ tr\ v_4$ = 5	Regions 2–3 = 0.56
2–4	$+\ +\ b\ +\ +$ = 15	Regions 2–4 = 1.00
3–4	$lg_1\ gl_2\ b\ tr\ v_4$ = 7	Regions 3–4 = 1.14
3–4	$+\ +\ +\ +\ +$ = 15	
1–2–3	$lg_1\ +\ b\ tr\ +$ = 5	
1–2–3	$+\ gl_2\ +\ +\ v_4$ = 6	
1–2–4	$lg_1\ +\ b\ +\ +$ = 1	
1–2–4	$+\ gl_2\ +\ tr\ v_4$ = 3	
1–3–4	$lg_1\ +\ +\ +\ +$ = 1	
1–3–4	$+\ gl_2\ b\ tr\ v_4$ = 0	
2–3–4	$lg_1\ gl_2\ +\ +\ +$ = 0	
2–3–4	$+\ +\ b\ tr\ v_4$ = 0	
1–2–3–4	$lg_1\ +\ b\ tr\ v_4$ = 0	
1–2–3–4	$+\ gl_2\ +\ +\ +$ = 1	
	Total = 1582	

the data on the linkage relationships in this reciprocal translocation were obtained from crosses where translocation individuals were used as the male parent. All of the recombination values reported in this paper, with the exception of table 2, are from crosses where the male parent was the heterozygote. The available data from reciprocal crosses for other

regions suggests that such a difference exists, i.e., the percentages of recombination are lower in the megasporocytes than in the microsporocytes of individuals carrying the translocation. No consistent differences were found in the reciprocal crosses of normal plants.

There is a considerable number of $2n + 1$ individuals in the progeny of a cross when plants heterozygous for the translocation are used as the female parent but these $2n + 1$ individuals cannot account for the differences in crossing-over in the reciprocal crosses.

TABLE 2

BACKCROSS DATA FROM PLANTS HETEROZYGOUS FOR THE TRANSLOCATION AND *pr* AND *bt*$_1$ IN NO. 5 AND *b* IN NO. 2. THE DATA LISTED UNDER THE HEADING "FEMALE" WERE OBTAINED WHEN HETEROZYGOUS INDIVIDUALS WERE USED AS THE FEMALE PARENT AND THE DATA UNDER THE HEADING "MALE" CAME FROM THE RECIPROCAL CROSS.

REGIONS		FEMALE	MALE	TOTAL
0	+ + *tr* + −	354	180	534
0	*pr bt* + *b* −	286	188	474
1	+ *bt* + *b* −	28	38	66
1	*pr* + *tr* + −	46	43	89
2	+ + + *b* −	1	2	3
2	*pr bt tr* + −	2	1	3
3	+ + *tr b* −	84	59	143
3	*pr bt* + + −	69	59	128
1–2	+ *bt tr* + −	0	1	1
1–2	*pr* + + *b* −	1	4	5
1–3	+ *bt* + + −	10	10	20
1–3	*pr* + *tr b* −	20	15	35
2–3	+ + + + −	2	0	2
2–3	*pr bt tr b* −	4	1	5
1–2–3	+ *bt tr b* −	1	0	1
1–2–3	*pr* + + + −	1	1	2
		909	602	1511
pr-bt		11.8%	18.6%	
bt-tr		1.3%	1.7%	
b-tr		21.0%	24.1%	
pr-tr		13.1%	20.3%	
pr-b		34.1%	44.4%	
bt-b		22.3%	25.8%	

As the writer pointed out in an earlier paper[3] there is an absence of interference in crossing-over for two of the four arms of the quadripartite figure of the reciprocal translocation. Additional data have since been gathered which support this conclusion and the data in table 2 show the absence of interference for the *pr-tr-b* regions. It would be, of course, desirable to study interference in all four arms of the translocation but this has not yet been possible.

Dobzhansky[9] has shown in Drosophila that there is a marked reduction

in crossing-over for that arm of the chromosome in which the translocation or interchange occurred. It should be possible to test for a similar reduction of crossing-over in this reciprocal translocation if the position of the point of breakage lies between two or more genes which are situated in the same arm of the chromosome. The linear order in No. 2 chromosome has been shown to be lg_1-gl_2-b-ts_1-insertion region-point of translocation-v_4. Since v_4 is the only factor in the same arm as the point of translocation an exact determination of the effect of the translocation on crossing-over is not possible. Yet the recombination values of gl_2 and b with v_4 should be different in translocation and control individuals since v_4 lies in the same arm as the break and some distance away from the insertion region. The amount of recombination between gl_2 and v_4 in translocation individuals was 32.9 per cent while the control crosses gave 42.5 per cent. More than a thousand individuals were classified in both of the two types of crosses so the difference is significant. The more critical comparison of the amount of recombination in the b-v_4 region was not obtained since the control data for the backcross of plants heterozygous for lg_1 gl_2 b v_4 were from seedling counts and an accurate classification of B and b was impossible. However, the data on the gl_2-v_4 region indicate that a reduction of crossing-over does occur in the arm of the No. 2 chromosome in which the break occurred.

Arguments have been presented for believing that v_2 pr bm_1 and bt_1 all lie to one side of the point of translocation in No. 5. Since bm_1 is situated in the short arm of No. 5 and with some certainty in the region between the insertion and the point of translocation, no reduction, or very little since bm_1 is near the insertion, would be expected in the percentage of recombination between pr-bt_1 or pr-bm_1 in translocation individuals as compared with the controls if pr was in the long arm of No. 5. No marked differences were found. The amounts of recombination found between different loci in No. 2 and No. 5 chromosomes in translocation and normal individuals are given below:

	TRANSLOCATION	CONTROL
lg_1-gl_2	21.2 per cent	21.2 per cent
gl_2-v_4	32.9 per cent	42.5 per cent
lg_1-v_4	45.7 per cent	50.2 per cent
b-ts_1	21.0 per cent	23.0 per cent
lg_1-b	35.0 per cent	34.0 per cent
b-v_4	24.2 per cent	
v_2-pr	46.3 per cent	42.0 per cent
pr-bt_1	18.6 per cent (male)	17.0 per cent
pr-bm_1	30.0 per cent	29.1 per cent

* Paper No. 201 from the Department of Plant Breeding, Cornell University, Ithaca, New York.

** The ratio of the two arms of the No. 2 chromosome found in the writer's cultures is 1.0:1.4 but McClintock has found certain strains of maize where the ratio is approximately 1.0:1.2.

[1] McClintock, Barbara, *Science*, **69**, 629 (1929).
[2] Burnham, C. R., *Proc. Nat. Acad. Sci.*, **16**, 269–277 (1930).
[3] Rhoades, M. M., *Ibid.*, **17**, 694–698 (1931).
[4] Burnham, C. R., *Proc. Sixth Int. Cong. Genetics*, **2**, 19–20 (1932).
[5] McClintock, Barbara, *Ibid.*, **2**, 126–128 (1932).
[6] McClintock, Barbara, *Missouri Agr. Exp. Sta. Bull.* **163** (1931).
[7] McClintock, Barbara, *Proc. Nat. Acad. Sci.*, **18**, 677–681 (1932).
[8] Rhoades, M. M., *Genetics*, **18**, 535–555 (1933).
[9] Dobzhansky, Th., *Ibid.*, **15**, 347–399 (1930).

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*A SECONDARY TRISOME IN MAIZE**

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Communicated November 7, 1933

Three types of trisomes have been identified in Datura by the cytogenetic investigations of Blakeslee, Belling and others. The primary trisomes are those in which a member of the monoploid set is represented in triplicate rather than in duplicate as is the case for the remainder of the set. Primary trisomes have been isolated in Datura, Zea, Lycopersicum, Matthiola, Oenothera and other plants and in Drosophila melanogaster. The three homologous chromosomes in a primary trisome give characteristic configurations at pachytene, diakinesis and metaphase I. A closed ring of three at diakinesis should never be found in a primary trisome since the two ends in an open V or chain of three are not homologous.

A secondary trisome differs from a primary trisome in that the extra chromosome is not a replicate of one of the members of the monoploid set but has in some way become modified so that its two ends are homologous. The conclusion that the two ends are homologous was reached from a study of the configurations at diakinesis and metaphase I (Belling and Blakeslee[1]) where a ring of three or a bivalent plus a closed univalent was commonly found. No adequate studies of the prophase synapsis have ever been made in secondary trisomes so Belling's conjecture that the extra chromosome is a double half chromosome has lacked convincing support. Secondary trisomes have been found previously only in Datura although Philp and Huskins[2] may have a modified secondary in Matthiola.

A tertiary trisome is one in which the supernumerary chromosome is composed in part of one member of the monoploid set and in part of a different member of the monoploid set. Tertiaries have been found in

Datura, Zea and possibly other forms. Tertiary trisomes give a range of configurations at diakinesis which is in accord with the theory that only homologous ends of chromosomes are attached. For a complete discussion of the types of configurations at diakinesis and metaphase I in primary, secondary and tertiary trisomes see Belling.[3]

This paper is concerned with the first secondary trisome which has been found in maize. Among the progeny from a selfed plant trisomic for chromosome 6 there occurred a variant which was strikingly different from its sibs. It resembled as a seedling some of the dwarf types of maize since it had an extremely stocky appearance with broad leaves which had blunt points. Somatic counts of root tips showed 21 chromosomes but the plant possessed only two satellited chromosomes so it probably was not trisomic for chromosome 6. Plants which are trisomic for chromosome 6 cannot be distiuguished from disomic sibs. As the plants matured the differences between the variant and its sibs increased. At maturity

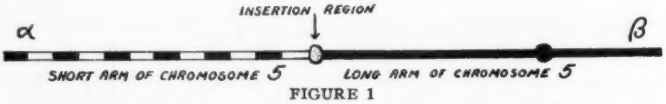


FIGURE 1

Diagrammatic representation of morphology of chromosome 5 in Zea as determined from pachytene measurements.

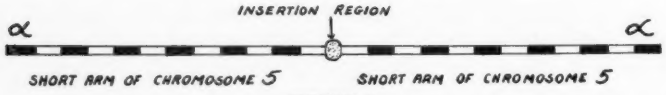


FIGURE 2

Diagrammatic representation of morphology of the modified chromosome in the secondary trisome.

the variant was of reduced stature, had thick leaves which were leathery in texture, and broad with extremely blunt points. The leaves were so stiff that they had no tendency to assume a pendant position near their tips. The tassel was not noticeably different from normal ones in appearance, although the presence of smut made this uncertain. This variant proved, when studied cytologically, to be a secondary trisome of chromosome 5.

Plants trisomic for chromosome 5 differ markedly from their disomic sibs in several ways. Their leaves are broader and have blunter tips than do the leaves of disomes. They tend to remain stiff and do not droop. The tassel of a trisome is more compact and sturdier in appearance. The stature of trisomes is less, giving them a stocky growth habit. They are several days later in flowering than their disomic sibs. Trisomic individuals as seedlings can usually be distinguished from disomes by the

broadness of the leaves and the bluntness of the tips, together with a somewhat reduced stature. The general difference in appearance and growth habit is so pronounced that an accurate classification of a segregating culture of mature plants into trisomic and disomic types can be made.

It is of interest to note that certain of the characteristics which distinguish the No. 5 primary trisomes were exaggerated in the secondary. This exaggeration of certain characters in the secondary is essentially

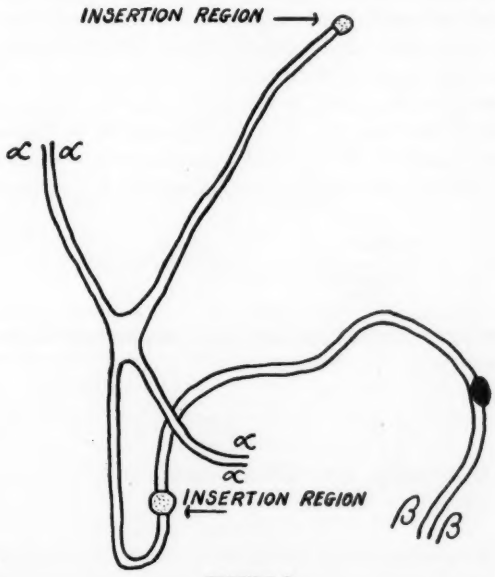


FIGURE 3

A camera lucida sketch at pachytene representing the synaptic relationships of two normal chromosomes 5 and the modified or secondary chromosome. The prominent knob in the long arm is in a heterozygous condition. This is one of the two types of pachytene association which were observed.

in agreement with the extensive work of Blakeslee on secondary trisomes in Datura. Knowing the characteristics of the primary and of one of the secondaries it should be possible to predict in advance the characteristics of the other secondary. Blakeslee accomplished this unique feat for one of the secondaries in Datura.

Chromosome 5 in Zea has an insertion region which is nearly median so that the two arms are nearly equal in length. The relative lengths of

the two arms from measurements at pachytene is 1.0:1.1. The longer arm carries in many strains of maize a large prominent knob. The morphology of chromosome 5 is shown diagrammatically in figure 1. One of the two chromosomes 5 present in the secondary trisome had the prominent knob in the long arm while the other did not. The morphology of the extra chromosome present in the secondary trisome is shown diagrammatically in figure 2. As the figure shows, the short arm of chromosome 5 is in duplicate and the insertion region is median.

Cytological studies of the secondary trisome were made at pachytene, diakinesis and metaphase I. The pachytene figures were the more illuminating since they made possible an exact determination of the morphology of the modified chromosome. At pachytene only two types of synaptic configurations were found, although other types may not have been detected. The first type is that represented in figures 3 and 3*a*. The insertion region of the modified chromosome occupies a terminal position with its two homologous arms synapsed for some distance. At

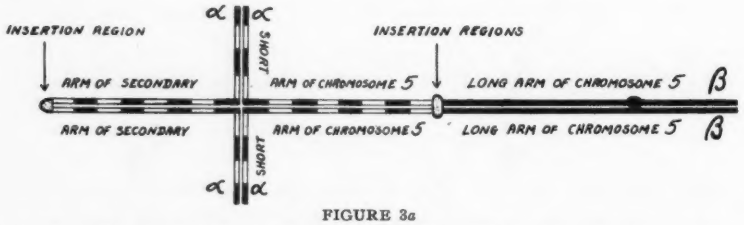


FIGURE 3*a*

A schematic representation of the synaptic relations of the three chromosomes in figure 3.

a variable distance from the terminal insertion region the two arms of the modified chromosome cease their synaptic relationship with each other and pair, individually, with the short arms of the two normal chromosomes 5 to give a cross-shaped figure which resembles those found in heterozygous translocations in Zea. Twelve clear figures of this type of pachytene association were found but the position of the center of the cross varied considerably. In one figure the distance from the center of the cross to the terminal insertion of the modified chromosome was one-third of the length of the entire arm while in another figure the center of the cross was approximately three-fourths the length of the arm from the terminal insertion. The rest of the figures showed the center of the cross occurring between these two observed extremes. Burnham[4] and McClintock[5] have observed that in translocation figures the position of the center of the cross varies and interpreted it as the pairing of non-homologous parts. It is important to point out that in this secondary trisome

the varied positions of the center of the cross is not caused by non-homologous association as the two arms of the modified chromosome are homologous throughout their lengths with the short arms of the two chromosomes 5.

The other observed type of pachytene association was that in which the two normal chromosomes 5 had synapsed completely leaving the modified chromosome unpaired. In these figures, and they were seen more frequently than the first type since they were easier to find, the unpaired modified chromosome always had a terminal insertion with its two arms pairing evenly with one another to the distal end of the chromosome. Figure 4 represents the unpaired modified chromosome and the two syn-

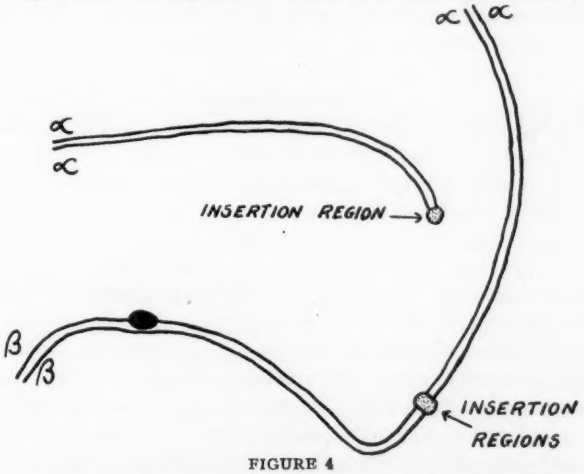


FIGURE 4

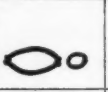
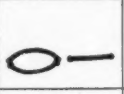
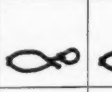
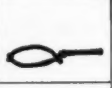
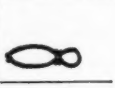
A camera lucida sketch of the second type of pachytene association. The two normal chromosomes 5 have paired together and the modified or secondary chromosome is left as a univalent. Its two homologous arms have synapsed leaving the median insertion region in a terminal position.

apsed normal chromosomes at pachytene. All of the cytological observations support the belief that the two arms of the modified chromosome were of equal length. Measurements of the first type of synaptic configuration at pachytene show that the distance from the terminal insertion region of the modified chromosome to the ends of its two arms is equal to the distance from the insertion regions of the normal chromosomes 5 to the end of their short arms. Likewise the measurements of the second class of configurations agree with the conclusion that the modified chromosome is composed of two entire short arms of chromosome 5 and has a median insertion region.

The types of configurations found at diakinesis are shown in table 1. The two commonest types were a ring of three, and a bivalent plus a closed univalent or circlet. Types 2 and 3 in table 1 undoubtedly arose from the pachytene configuration having a bivalent composed of the two normal chromosomes 5 with the modified chromosome unpaired. The remaining types in table 1 probably arose from the first type of pachytene association with the failure to form, or the breakage of, chiasmata giving the range of figures found at diakinesis.

TABLE 1

DIFFERENT TYPES OF CONFIGURATIONS FOUND IN THE SECONDARY TRISOME AT DIAKINESIS. THE FREQUENCY WITH WHICH EACH OF THE DIFFERENT TYPES OCCURRED IS INDICATED BY THE NUMBER BELOW IT IN THE TABLE.

TYPE 1	TYPE 2	TYPE 3	TYPE 4	TYPE 5	TYPE 6	TYPE 7
						
27	20	5	7	1	2	0

We will assume that a ring of three at diakinesis arises from a pachytene association of the first type in which at least three chiasmata are present. One of the chiasmata occurs in the long arms of the two chromosomes 5 and when it is terminalized will result in the ends of the two long arms being associated at diakinesis. A second chiasma occurs in that region between the center of the cross and the distal end of one normal chromosome 5 and one arm of the modified chromosome. The third chiasma occurs beyond the center of the cross between the short arm of the other normal chromosome 5 and the other arm of the modified chromosome. When these two chiasmata terminalize they will presumably result in the two arms of the modified chromosome being associated with the two short arms of the two normal chromosomes 5. The three chiasmata should then give a ring of three at diakinesis, and the observed frequency of rings was high. The pachytene figures clearly showed that the distance from the insertion regions to the center of the cross is, on the average, at least half the length of the short arm. There is no reason to suppose that the formation of chiasmata in the two regions from the center of the cross to the insertion of the modified chromosome or to the insertions of the two normal chromosomes is completely inhibited. The occurrence of chiasmata in the regions between the center of the cross and the distal ends of the short arms would, from analogy of genetic interference, tend to reduce the frequency with which chiasmata would be formed coinci-

dentally in the regions between the center of the cross and the insertions. The pairing at pachytene is of equal intimacy throughout the figure and the considerable distance from the insertions to the center of the cross would surely seem to permit the formation of chiasmata unless these regions in some way are restricted to few chiasmata. It is true that the genetic and cytological maps for the two autosomes in Drosophila melanogaster indicate that the amount of crossing-over per unit of physical length is low near the spindle fibre. We have some reason for believing that such a condition may hold for chromosome 2 in Zea but the evidence is not wholly convincing.** The genetic map of the long arm of chromosome 5 is at least 80 units long and the but-slightly shorter arm should have approximately the same length of genetic map so a large proportion of the figures should have two or more chiasmata in the short arm. The occurrence of a chiasma in either the region between the insertions of the two chromosomes 5 and the center of the cross or in the region between the center of the cross and the insertion of the modified chromosome coincidentally with the three chiasmata in the other three regions of the figure should give a figure eight (type 7 of table 1) at diakinesis. The most striking feature of the diakinesis configurations was the failure to find any figure eights. The number of observed figures in table 1 is small but many more cells were examined during the summer in an endeavor to find such a type of configuration. None was found, although it may be harder to detect and, therefore, overlooked. Unfortunately some of the temporary smears spoiled before counts could be made for the different types of figures present. The data in table 1 are, however, in the writer's opinion, a random sample and a fair representation of the types of figures found at diakinesis. The failure to find figure eights at diakinesis can be interpreted in two ways. Either no chiasmata are formed between the insertion regions and the center of the cross coincidentally with chiasmata in the other regions or else they are broken or cancelled at diplotene. It seems more probable that the rare, at best, occurrence of figure eights can be explained by the breaking or cancellation of chiasmata at diplotene. This conclusion would support Sax's theory of crossing-over and would be opposed to that of Darlington.

The only genetic factor in the fifth linkage group in maize whose location is with certainty known to be in the short arm of chromosome 5 is bm_1 (McClintock[6]) and the position of this gene is close to the insertion region. But certain of the characteristics which distinguish the primary trisome of chromosome 5 must be assigned to genetic factors which are situated in the short arm of the chromosome.

* Paper No. 202 from the Department of Plant Breeding, Cornell University, Ithaca, New York.

** See preceding paper by the writer in this Journal.

[1] Belling, John, and Blakeslee, A. F., *Proc. Nat. Acad. Sci.*, **10**, 116–120 (1924).
[2] Philp, J., and Huskins, C. L., *J. Genet.*, **24**, 359–404 (1931).
[3] Belling, John, *J. Genet.*, **18**, 177–205 (1927).
[4] Burnham, C. R., *Proc. Sixth Int. Cong. Genet.*, **2**, 19–20 (1932).
[5] McClintock, Barbara, *Zeit. Zellforsch. mik. Anat.*, **19**, 191–237 (1933).
[6] McClintock, Barbara, *Proc. Nat. Acad. Sci.*, **18**, 677–681 (1932).

THE COLOR CHANGES OF ELASMOBRANCH FISHES

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Communicated October 28, 1933

Very little work has been done on the color changes of elasmobranch fishes. In 1921 Schaefer reported in an appendix to an extended article on the color changes in flatfishes that the skates Raja clavata and Raja batis showed on being tested no such changes. A little over a decade later Lundstrom and Bard (1932) described striking color changes in the dogfish Mustelus canis and showed that in this fish the secretions from the pituitary gland were accountable for the dark phase of this fish. The light phase of Mustelus was studied in 1933 by Parker and Porter who in a paper now in press (1934) have demonstrated that this coloration is induced by the direct action of nerves. In the course of the work on dogfishes opportunity was found to test the possibility of color changes in the common skate Raja erinacea which was available in the laboratory at that time.

Two individuals, indistinguishably dark brown, were placed one in a white-walled sea-water tank illuminated from above and the other in a similar black-walled tank. After eighteen hours the skate in the white tank was light brown and that in the black tank was dark brown. The fishes were then transferred each to the other tank. After the light skate had been in the dark tank nine hours it became decidedly dark, but the dark skate after an equal interval in the light tank remained dark. Twelve hours later, however, this fish had become very light and of a hue that could be described as pinkish white. At this stage in the tests unfortunately the mate of this light fish died.

The light fish, now extremely pale in tint, was transferred to the black tank. In two hours it had darkened considerably and in twelve hours it was again dark brown. In consequence of the need of the tanks for other experimental work, tests on this skate had to be discontinued. Enough, however, had been observed on these two fishes to justify the conclusion that Raja erinacea, like Mustelus canis, is subject to reasonably striking color changes. From this standpoint it would seem desirable

to retest the European forms previously reported on by Schaefer to ascertain whether upon close examination they, too, might not show color changes.

[1] Contribution No. 37.

Lundstrom, H. M., and Bard, P., "Hypophysial Control of Cutaneous Pigmentation in an Elasmobranch Fish," *Biol. Bull.*, **42**, 1–9 (1932).

Parker, G. H., and Porter, H., "The Control of the Dermal Melanophores in Elasmobranch Fishes," *Ibid.* (in press) (1934).

Schaefer, J. G., "Beiträge zur Physiologie des Farbenwechsels der Fische. I. Untersuchungen an Pleuronectiden," *Arch. ges. Physiol.*, **188**, 25–48 (1921).

ON OVERLAP

BY EDWIN B. WILSON

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Communicated November 14, 1933

In "Abilities of Man" Spearman[1] pictures overlap between specific abilities by overlapping areas somewhat analogous to the famous Euler circles of logic. From the testing point of view one might think of overlap as arising if questions from a test for one ability were found by accident or by intent upon a test for some other ability. The analytical resolution of a test into the general and a specific as $\mathbf{a} = r_{ag}\mathbf{g} + \sqrt{1 - r_{ag}^2}\mathbf{s}_a$ with the condition of non-correlation (orthogonality) imposed on $\mathbf{g}, \mathbf{s}_a, \mathbf{s}_b, \ldots$ suggests that overlap is presumably a vectorial rather than a scalar phenomenon. As, however, the condition for overlap is that the tetrads do not vanish[2] and as the tetrads taken as a group involve all the tests, it might seem that the question as to overlap or non-overlap became one involving the mutual relations of all the tests, a sort of tensorial phenomenon.

If we had in advance non-correlating tests for $\mathbf{g}, \mathbf{s}_a, \mathbf{s}_b, \ldots$ any test $\mathbf{u}$ could be written at once as[3]

$$\begin{aligned}\mathbf{u} &= \mathbf{u}.\mathbf{g}\mathbf{g} + \mathbf{u}.\mathbf{s}_a\mathbf{s}_a + \mathbf{u}.\mathbf{s}_b\mathbf{s}_b + \ldots \\ &= r_{ug}\mathbf{g} + r_{us_a}\mathbf{s}_a + r_{us_b}\mathbf{s}_b + \ldots,\end{aligned}$$

the components of $\mathbf{u}$ along the orthogonal vectors $\mathbf{g}, \mathbf{s}_a, \mathbf{s}_b, \ldots$ would be known and any residual component of $\mathbf{u}$ would be orthogonal to $\mathbf{g}, \mathbf{s}_a, \mathbf{s}_b, \ldots$ and dependent on further specific factors. It is, however, supposed that $\mathbf{g}, \mathbf{s}_a, \mathbf{s}_b, \ldots$ are not given in advance but are to be determined (so far as determinable) by the intercorrelation analysis of $\mathbf{a}, \mathbf{b}, \ldots$. If there be three tests the question of overlap between their specifics does not arise because three tests (given that certain inequalities are satisfied) are always resoluble into one general and three specifics.

The simplest case therefore for which one can discuss overlap is when there are four tests **a, b, c, d** and not all the three tetrads $\mathbf{a} \times \mathbf{b.c} \times \mathbf{d}$, $\mathbf{c} \times \mathbf{a.b} \times \mathbf{d}$, $\mathbf{b} \times \mathbf{c.a} \times \mathbf{d}$ vanish. As there is a linear relation

$$\mathbf{a} \times \mathbf{b.c} \times \mathbf{d} + \mathbf{c} \times \mathbf{a.b} \times \mathbf{d} + \mathbf{b} \times \mathbf{c.a} \times \mathbf{d} = 0$$

between the three, not more than one of the tetrads can vanish and in general two will be of one sign and the third of the other. The tetrads as a group, taken with their negatives, are six quantities symmetric in the four elements **a, b, c, d** in the sense that the group as a whole is not changed by any permutation of the four letters. Hence the condition for overlap or for non-overlap is symmetrical in the four tests—a symmetry that may be stated geometrically by saying that the 4-gon on the three-dimensional surface of a sphere in four dimensions has or has not an orthocenter. Thus it is not a question of overlap of two specifics but of a condition of overlap existing or not existing among four.

It is always possible to assume asymmetrically that there is no overlap between any given set of three, say $\mathbf{s}_a$, $\mathbf{s}_b$, $\mathbf{s}_c$, namely, that

$$\mathbf{a} = r_{ag}\mathbf{g} + \sqrt{1 - r_{ag}^2}\,\mathbf{s}_a,\ \mathbf{b} = r_{bg}\mathbf{g} + \sqrt{1 - r_{bg}^2}\,\mathbf{s}_b,\ \mathbf{c} = r_{cg}\mathbf{g} + \sqrt{1 - r_{cg}^2}\,\mathbf{s}_c$$

and to throw all the overlap on the fourth, **d**. Then suppose

$$\mathbf{d} = r_{dg}\mathbf{g} + x\,\mathbf{s}_a + y\,\mathbf{s}_b + z\,\mathbf{s}_c + w\,\mathbf{s}_d,\quad r_{dg}^2 + x^2 + y^2 + z^2 + w^2 = 1;$$

$$\mathbf{b} \times \mathbf{c} = r_{bg}\sqrt{1 - r_{cg}^2}\,\mathbf{g} \times \mathbf{s}_c - r_{cg}\sqrt{1 - r_{bg}^2}\,\mathbf{g} \times \mathbf{s}_b + \sqrt{1 - r_{bg}^2}\,\sqrt{1 - r_{cg}^2}\,\mathbf{s}_b \times \mathbf{s}_c$$

$$\mathbf{a} \times \mathbf{d} = (xr_{ag} - r_{dg}\sqrt{1 - r_{ag}^2})\,\mathbf{g} \times \mathbf{s}_a + yr_{ag}\mathbf{g} \times \mathbf{s}_b + zr_{ag}\mathbf{g} \times \mathbf{s}_c + wr_{ag}\mathbf{g} \times \mathbf{s}_d + y\sqrt{1 - r_{ag}^2}\,\mathbf{s}_a \times \mathbf{s}_b + z\sqrt{1 - r_{ag}^2}\,\mathbf{s}_a \times \mathbf{s}_c + w\sqrt{1 - r_{ag}^2}\,\mathbf{s}_a \times \mathbf{s}_d.$$

Then the tetrad values are three like

$$t_1 = \mathbf{b} \times \mathbf{c.a} \times \mathbf{d} = zr_{ag}r_{dg}\sqrt{1 - r_{cg}^2} - yr_{ag}r_{cg}\sqrt{1 - r_{bg}^2}.$$

Of these three equations only two are independent. We have also

$$r_{ad} = r_{ag}r_{dg} + x\sqrt{1 - r_{ag}^2},\ r_{bd} = r_{bg}r_{dg} + y\sqrt{1 - r_{bg}^2}\,r_{cd} = r_{cg}r_{dg} + z\sqrt{1 - r_{cg}^2}.$$

Thus we may eliminate x, y, z by introducing the unknown r_{dg}. Then

$$t_1 = (r_{bd} - r_{bg}r_{dg})r_{ag}r_{cg} - (r_{cd} - r_{cg}r_{dg})r_{ag}r_{bg} = (r_{bd}r_{cg} - r_{cd}r_{bg})r_{ag}$$

and r_{dg} disappears. The condition that **d** be a unit vector becomes a quadratic in w and r_{dg}, neither of which is determined but one of which is determined if the other is known, namely,

$$w^2 = 1 - \frac{r_{ad}^2}{1 - r_{ag}^2} - \frac{r_{bd}^2}{1 - r_{bg}^2} - \frac{r_{cd}^2}{1 - r_{cg}^2}$$
$$+ 2r_{dg}\left[\frac{r_{ad}r_{ag}}{1 - r_{ag}^2} + \frac{r_{bd}r_{bg}}{1 - r_{bg}^2} + \frac{r_{cd}r_{cg}}{1 - r_{cg}^2}\right]$$
$$- r_{dg}^2\left[1 + \frac{r_{ag}^2}{1 - r_{ag}^2} + \frac{r_{bg}^2}{1 - r_{bg}^2} + \frac{r_{cg}^2}{1 - r_{cg}^2}\right].$$

It is of course necessary that w^2 should be positive.

With any value of r_{dg} which makes w^2 positive the expression for $\mathbf{d}$ becomes

$$\mathbf{d} = r_{dg}\mathbf{g} + \frac{r_{ad} - r_{ag}r_{dg}}{\sqrt{1 - r_{ag}^2}}\mathbf{s}_a + \frac{r_{bd} - r_{bg}r_{dg}}{\sqrt{1 - r_{bg}^2}}\mathbf{s}_b + \frac{r_{cd} - r_{cg}r_{dg}}{\sqrt{1 - r_{cg}^2}}\mathbf{s}_c + w\mathbf{s}_d.$$

Owing to the independence of r_{dg}, $\mathbf{d}$ may have a variety of amounts of $\mathbf{g}$ and a variety of amounts of $\mathbf{s}_a$, $\mathbf{s}_b$, $\mathbf{s}_c$, $\mathbf{s}_d$ for given values of the tetrads. Thus if the tetrads do not vanish and if we postulate that there is a general $\mathbf{g}$, the problem of what the component of $\mathbf{d}$ is in $\mathbf{g}$, $\mathbf{s}_a$, $\mathbf{s}_b$, $\mathbf{s}_c$, $\mathbf{s}_d$ is indeterminate; there is no analytic solution to the overlap problem, because there are an infinity of solutions. It is observable, however, that in general there is overlap upon all the specifics. Moreover it is not $\mathbf{s}_d$ which overlaps on any of them unless we arbitrarily write

$$\sqrt{1 - r_{dg}^2}\,\mathbf{s}_d' = \mathbf{d} - r_{dg}\mathbf{g}$$

and obtain the expression for $\mathbf{s}_d'$ in terms of $\mathbf{s}_a$, $\mathbf{s}_b$, $\mathbf{s}_c$, $\mathbf{s}_d$; with r_{dg} unknown and in fact indeterminable this is a somewhat artificial procedure.

It has been assumed that there is one $\mathbf{g}$ and that the non-vanishing of the tetrads means overlap. This is what Spearman assumes. Kelley[4] would say that the non-vanishing of the tetrads means that there is more than one general factor. It may be observed that Kelley shows that the resolution of four tests into two generals and four specifics may be accomplished in an infinity of ways, much as the overlap problem may be solved in an infinity of ways.

One may proceed in a different manner. If the test $\mathbf{d}$ does not satisfy the tetrad equations one may modify it, i.e., replace it by a test $\mathbf{d}'$ which does. This problem also may be approached analytically. We may ask what test $\mathbf{d}'$ linearly dependent on $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ will make the tetrads vanish. The reason for restricting $\mathbf{d}'$ to linear dependence on $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ is to obviate the introduction of elements extraneous to those tests. Now the team

$$\mathbf{t} = \frac{\mathbf{a}}{\mathbf{b} \times \mathbf{a}.\mathbf{c} \times \mathbf{a}} + \frac{\mathbf{b}}{\mathbf{a} \times \mathbf{b}.\mathbf{c} \times \mathbf{b}} + \frac{\mathbf{c}}{\mathbf{a} \times \mathbf{c}.\mathbf{b} \times \mathbf{c}}$$

derived from **a, b, c**, though in this form not a unit vector, has the property that its tetrads formed with **a, b, c** all vanish. As the tetrads are linear in **d**, their values for **d** and for $\mathbf{d} + x\mathbf{t}$ will be the same and hence if any test $\mathbf{d}'$ is found, $\mathbf{d}' + x\mathbf{t}$, where x is arbitrary except for such restrictions on its range as may be necessary to satisfy certain inequalities, will also be a solution. Thus if there is any solution for $\mathbf{d}'$ there are an infinite number.

It is possible to write down a vector $\mathbf{d}'$, not a unit vector though easily adjustable to such by the usual procedure, which makes the tetrads vanish. Take

$$\mathbf{d}' = \mathbf{d} - \frac{\mathbf{a} \times \mathbf{b.c} \times \mathbf{d}}{\mathbf{a} \times \mathbf{b.c} \times \mathbf{d}}\,\mathbf{b} - \frac{\mathbf{a} \times \mathbf{c.b} \times \mathbf{d}}{\mathbf{a} \times \mathbf{c.b} \times \mathbf{c}}\,\mathbf{c}.$$

It is easily verified that the tetrads do vanish. Thus

$$\mathbf{c} \times \mathbf{a.b} \times \mathbf{d}' = \mathbf{c} \times \mathbf{a.b} \times \mathbf{d} - \frac{\mathbf{a} \times \mathbf{c.b} \times \mathbf{d}}{\mathbf{a} \times \mathbf{c.b} \times \mathbf{c}}\,\mathbf{c} \times \mathbf{a.b} \times \mathbf{c} = 0,$$

and so for $\mathbf{a} \times \mathbf{b.c} \times \mathbf{d}'$, while

$$\mathbf{b} \times \mathbf{c.a} \times \mathbf{d}' = \mathbf{b} \times \mathbf{c.a} \times \mathbf{d} - \frac{\mathbf{a} \times \mathbf{b.c} \times \mathbf{d}}{\mathbf{a} \times \mathbf{b.c} \times \mathbf{b}}\,\mathbf{b} \times \mathbf{c.a} \times \mathbf{b} - \frac{\mathbf{a} \times \mathbf{c.d} \times \mathbf{d}}{\mathbf{a} \times \mathbf{c.b} \times \mathbf{c}}\,\mathbf{b} \times \mathbf{c.a} \times \mathbf{c} = \mathbf{b} \times \mathbf{c.a} \times \mathbf{d} + \mathbf{a} \times \mathbf{b.c} \times \mathbf{d} + \mathbf{c} \times \mathbf{a.b} \times \mathbf{d} = 0.$$

The expression for $\mathbf{d}'$ might of course be made symmetric in **a, b, c**, but with a loss of apparent analytical simplicity, by adding an appropriate multiple of **t**. It may be observed that the infinity of solutions $\mathbf{d}' + x\mathbf{t}$ will in general have components along **a** and **b** and **c**. Hence if for any possible value of x, the value $r_{d'g}$ be obtained and $\mathbf{d}'$ be written as

$$\frac{\mathbf{d}'}{\sqrt{\mathbf{d}'.\mathbf{d}'}} = r_{d'g}\mathbf{g} + \sqrt{1 - r^2_{d'g}}\,\mathbf{s}_{d'}$$

the value of **d** may be expressed in terms of $\mathbf{g}, \mathbf{s}_a, \mathbf{s}_b, \mathbf{s}_c, \mathbf{s}_{d'}$ with all four of the specific present in general.

Following the geometric picture suggests a different though equivalent analysis. The vector

$$\mathbf{D} = \begin{vmatrix} 1 & r_{ab} & r_{ac} & r_{ad} \\ r_{ab} & 1 & r_{bc} & r_{bd} \\ r_{ac} & r_{bc} & 1 & r_{cd} \\ \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{d} \end{vmatrix} \quad \text{with } \Delta = \begin{vmatrix} 1 & r_{ab} & r_{ac} & r_{ad} \\ r_{ab} & 1 & r_{bc} & r_{bd} \\ r_{ac} & r_{bc} & 1 & r_{cd} \\ r_{ad} & r_{bd} & r_{cd} & 1 \end{vmatrix} > 0$$

is one which is perpendicular to **a, b, c** and such that **D.d** is positive. (If $\Delta < 0$ we should change the sign of **D.**) The vector $\mathbf{d}'' = \mathbf{D}/\sqrt{A_d \Delta}$, where A_d is the minor in **D** corresponding to the element **d**, is of unit magnitude. As $\mathbf{a.d}'' = \mathbf{b.d}'' = \mathbf{c.d}'' = 0$, the tetrads for **a, b, c, d**″ all vanish. If now the vector **t** representing the team due to **a, b, c** and of unit magnitude (which differs from the **t** written above only by a factor) be combined with **d**″ to obtain

$$\mathbf{d}''' = \mathbf{d}'' \sin\theta + \mathbf{t}\cos\theta$$

we have in **d**‴ a unit vector which contains a parameter θ and which for any value of θ will make the tetrads vanish.

If $\theta = 90°$ we have $\mathbf{d}''' = \mathbf{d}''$. The solution of **a, b, c, d**‴ for **g** will then give $\mathbf{d}'''.\mathbf{g} = 0$ and **d**‴ will reduce to $\mathbf{s}_{d'''}$, a pure specific independent of the general **g.** If, however, we take $\cos\theta = r_{tg}$, the vector **d**‴ will have the property

$$\mathbf{a.d}''' = \mathbf{a.}tr_{tg} = \mathbf{a.g} \text{ and } \mathbf{a.d}''' = \mathbf{a.g}\ \mathbf{d}'''.\mathbf{g}$$

so that $\mathbf{d}'''.\mathbf{g} = 1$, and the vector **d**‴ will be precisely the general factor **g.** Hence if we have a test **d** for which the tetrads do not vanish we may take such linear combinations of **d** with **a, b, c** as will give analytically a pure specific or the pure general or a mixture of specific and general.

Thus although Piaggio or another, having given a set of tests, may throw away certain of them on the criterion of the tetrads; if one would keep a test which violates the criterion and proceed to try to discuss overlap he finds the problem indeterminate whereas if he would undertake to modify the test so as to remove overlap he finds the problem also indeterminate, and in either case the interpretation of the different solutions would appear to differ. Overlap is a very complex relationship between all the tests. It is difficult to believe that it can be removed by psychological procedures of building new tests without introducing a degree of arbitrariness analogous to the indetermination in the analytical solution. Indeed it is difficult to believe that so complex a notion as overlap is, in the absence of generals and specifics defined in advance, can be sensed with a definiteness which would guide the instincts of the psychologist in modifying tests which did not satisfy the tetrad criterion. The discovery that the batteries of tests actually administered do tend to satisfy the tetrad conditions was a noteworthy accomplishment and so was the interpretation of the phenomenon as indicative of the presumption that the tests really involved a common factor (the general factor). The complexities which the notion of overlap introduces may be an indication that a further major advance is needed, but they may also be an indication that the analysis has been pushed so far as to involve a welter of detail

which will limit the fruitfulness of attempts to advance along this particular path.

[1] C. Spearman, *Abilities of Man, Their Nature and Measurement*, Macmillan, 1927; see Appendix, p. viii.

[2] H. T. H. Piaggio, *Brit. J. Psychol.*, **24**, 88–105 (1933) states the matter clearly (p. 104) when in discussing Kelley's tests he says: "However, in deciding what tests to retain we make the tetrad relations the criterion, and *not* preconceived notions about dissimilarity of tests; indeed in the table below we retain three forms of memory test." Now unless memory be the general g, it would seem as though memory for words, memory for meaningful symbols and memory for meaningless symbols should have in common some general memory factor in addition to g, but they do not.

[3] For notation see E. B. Wilson, these PROCEEDINGS, **19**, 768, 882 (1933), and preceding papers.

[4] T. L. Kelley, *Crossroads in the Mind of Man: A Study of Differentiable Mental Abilities*, Stanford Univ. Press, 1928.

ON THE TEMPERATURE DEPENDENCE OF FERRO-MAGNETIC SATURATION

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1. Accurate data on the saturation intensity of magnetization at low temperatures for monocrystalline materials were recently published by Allen and Constant[1] who, moreover, show that their results fit into the following interesting law: Let the saturation intensity at the temperature T be denoted by M_{ST}, the same intensity at the absolute zero of temperature by M_{SO} and the Curie point by T_C. If the ratio M_{ST}/M_{SO} is plotted against T/T_C, the points for all ferro-magnetic crystals of the cubic system fall onto one and the same universal curve. Two other curves obtain in the case of hexagonal and of ortho-rhombic crystals.

The writer has given some time ago a theory of ferro-magnetism[2] which represents the continuation of previous theoretical work by Weiss,[3] Heisenberg[4] and Bloch[5] and which leads to precisely the above results, although it was carried through in detail only for the case of simple cubic crystals. It is, therefore, interesting to see whether the formula derived in this theory for the saturation intensity of magnetization as a function of temperature quantitatively fits the data now available. This is, in fact, the case: It will be shown in section 3 that a constant left indeterminate in the present form of the theory can be chosen so as to bring it into a very good agreement with the results of Allen and Constant.

2. As was shown in the paper referred to,[2] the intensity of magnetic saturation is obtained by partial differentiation of a function denoted by $Z^1(H)$:

$$M_{ST} = kT\left[\frac{\partial \log Z^1(H)}{\partial H}\right]_{H=0}, \tag{1}$$

where H is the magnetic strength of field and k the Boltzmann constant. If we introduce the abbreviation

$$\tau = 2\mu H/kT, \tag{2}$$

(μ being the Bohr magneton) there follows

$$M_{ST} = 2\mu\left[\frac{\partial \log Z^1(H)}{\partial \tau}\right]_{\tau=0}. \tag{3}$$

For the function $Z^1(H)$ we gave in Eq. (38) l. c. the expression

$$\log Z^1(H) = {}^1/_2 N\tau + \\ + \frac{8N}{\pi^3}\int\int\int_0^{\frac{\pi}{2}} \log\{1 + \exp[-\tau - 2\beta(\sin^2\xi + \sin^2\eta + \sin^2\zeta)]\} \\ d\xi d\eta d\zeta \tag{4}$$

where β is inversely proportional to the temperature

$$\beta = \Theta/T, \tag{5}$$

Θ being a constant.

The expression (4) can be transformed by expanding with respect to $\exp[-2\beta(\sin^2\xi + \sin^2\eta + \sin^2\zeta)]$ under the sign of the integral and remembering that

$$\frac{2}{\pi}\int_0^{\frac{\pi}{2}} \exp(n\beta\cos 2\xi)d\xi = J_0(in\beta), \tag{6}$$

where $J_0(x)$ denotes the Bessel function of the index 0 and of the argument x. We arrive, therefore, at the expression

$$\log Z^1(H) = {}^1/_2 N\tau - N\sum_{n=1}^{\infty}(-1)^n \exp(-n\tau - 3n\beta)[J_0(in\beta)]^3/n. \tag{7}$$

According to Eq. (3) we obtain for the saturation intensity

$$M_{ST} = M_{SO}\left\{1 + 2\sum_{n=1}^{\infty}(-1)^n[\exp(-n\beta)J_0(in\beta)]^3\right\}, \tag{8}$$

where $M_{SO} = N\mu$. The numerical evaluation of this formula leads to the table of values given in table 1.

For reasons which will be given in section 4, we have included in this table also some values of the function $\log Z^2(0)/N$ which were computed with the help of the formula

$$\log Z^2(0) = N[0.693 - 1.5\beta + 0.1875\beta^2 - 0.0293\beta^4 + \ldots]. \quad (9)$$

3. The characteristic temperature Θ which appears in our formulas and tables does not coincide with the Curie point T_C. The writer adduced general consideration (l. c.) which tended to show that Θ must be pro-

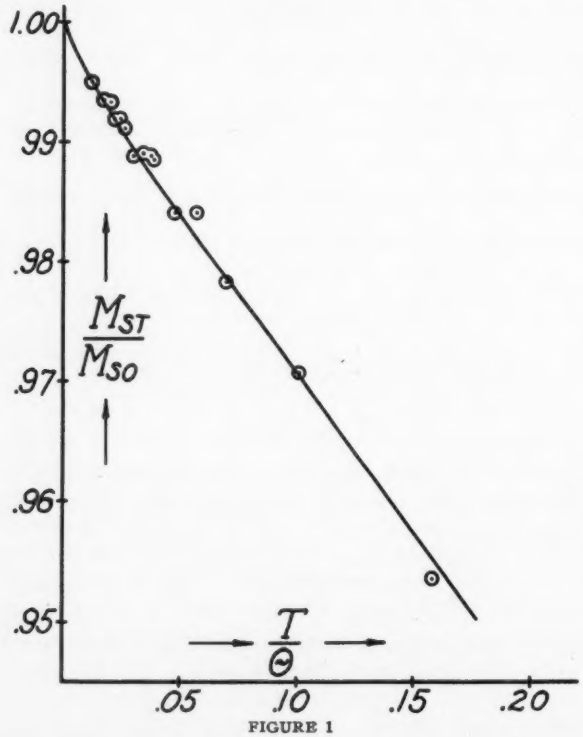

FIGURE 1

portional to T_C and even roughly estimated the factor of proportionality putting it at, approximately, $^2/_3$. It must be admitted, however, that this argument did not constitute a mathematical proof and that the theory in its present state does not permit to settle the question (compare section 4). It will be, therefore, better to regard Θ as an empirical constant and to ask whether the experimental evidence supports the view that it is proportional to the Curie point. In order to test this, we give

in figure 1 the curve which results from our formula (8) and from table 1 if we put $\Theta = 0.81\ T_C$. As in Allen and Constant's graph, M_{ST}/M_{SO} is plotted here against $(T/T_C)^2$ and the circles in our figure represent the experimental results of these authors within the accuracy with which we could infer them from their diagram.

It will be seen that the agreement is very satisfactory within the whole range of temperatures covered by the experimental data, i.e., up to 0.4 of the Curie point. Since our theory works with approximations which are justifiable for small T/Θ but become more and more questionable as the temperature rises, we cannot expect it to hold much above this range. Because of these limitations of the theory, it is unable to describe the conditions in the vicinity of the Curie point, so that the connection between Θ and T_C remains theoretically undeterminate. For this reason, the empirical confirmation of the proportionality between Θ and T_C receives a particular significance. There exists at present a general concurrence of opinion that the ferromagnetic substances consist of per-

TABLE 1

β	T/Θ	M_{ST}/M_{SO}	$\log Z^1(0)/N$	$\log Z^2(0)/N$
∞	0	1.000	0.000	
10	0.100	0.997		
8	0.125	0.995		
6	0.167	0.993		
5	0.200	0.990		
4	0.250	0.986	0.009	
3	0.333	0.978		
2.5	0.400	0.969		
2.00	0.500	0.953	0.026	
1.00	1.000	0.839	0.090	
0.90	1.111	0.811	0.106	
0.80	1.250	0.769	0.126	
0.60	1.667	0.668	0.187	−0.15
0.50	2.000		0.23	−0.012
0.40	2.500		0.28	0.12
0.20	5.000		0.44	0.40
0.00	∞		0.69	0.69

manently magnetized microcrystalline units or "blocks." The question arises, therefore, whether the disappearance of ferro-magnetic properties characterizing the Curie point is due to the sudden loss of permanent magnetization by these blocks (primary Curie point) or to the difficulty of aligning the permanently magnetized blocks above a certain temperature (secondary Curie point). The quantity M_{ST} is, simply, the intensity of permanent magnetization of the primary blocks and, according to the experimental results of Allen and Constant, it is determined by T_C even at very low temperatures. It is hard to see what the behavior of M_{ST}

in the neighborhood of the absolute zero can have to do with a secondary Curie point, and the conclusion must, therefore, be that in the case of the substances measured by these authors the temperature T_C refers to the primary Curie point of the blocks themselves, a result which can be strongly supported by other lines of reasoning.

The existence of another, higher Curie point was inferred by Forrer[6] from the form of the susceptibility curve above the lower, directly measurable Curie point and Bitter[7] interprets it as the secondary point in the above sense. If this interpretation is correct, the present theory is unable to explain the remarkable fact that Forrer's upper Curie point lies in all cases pretty close to the lower. However, in the writer's opinion, the existence of this upper Curie point is not beyond question because it is based on an extrapolation by means of a theoretical formula of uncertain validity. Only if the secondary Curie point happens to lie lower than the primary it can be observed in a direct and unambiguous way. Perhaps this case is realized for cobalt-iron alloys,[8] as suggested by Forrer.

It goes without saying that for crystals of the non-cubical system both the form of the law (8) and the connection between Θ and T_C will be different. We expect, however, that for low temperatures, the first change will be smaller than the second, so that the formula (8) will still retain a certain value as an approximation.[9]

4. The physical reason for the existence of a primary Curie point must be, of course, that for low temperatures the blocks are found mainly in quantic states associated with a large magnetic moment while, from a certain temperature on, the unmagnetized states have the higher probability. In our theory the criterion for this change of dominance is when the functions we denote by $Z^1(0)$ and $Z^2(0)$ become numerically equal. Using rather crude and inaccurate approximations for these functions, we estimated (l. c.) that this equality takes place at a temperature of the order of $3\Theta/2$ and that even the present form of the theory (which is admittedly inaccurate for temperatures that are not low) gives the Curie point almost quantitatively. We are compelled to take this statement back: It will be seen from table 1 that $Z^2(0)$ remains for all temperatures below $Z^1(0)$. This means that the terms neglected in the theory make themselves felt already at temperatures much below the Curie point, so that the decline of the theoretical curve of saturation intensities is considerably slower than that of the experimental.[10]

It will be useful to enumerate here the main simplifications which are contained in Bloch's and in our theory. Perhaps, the most important inaccuracy is that inherent in the use of Slater's equation which is derived by treating the antisymmetrical wave functions in determinant form, as if they were orthogonal, while, in reality, they are only approximately orthogonal. It is very hard to estimate the error introduced by this

omission and to say whether its correction will lead only to a new interpretation of the constants of our equations or to an actual change of their form. Another simplification is the neglect of polar and excited states: While there is reason to assume that their influence is small, its exact estimate is still lacking. Further inaccuracies were discussed in our preceding paper: The use of Bloch's periodicity condition in place of the actual border conditions, the restriction to interactions between adjacent atoms and to first order perturbations. With these omissions the mathematical side of the theory becomes simple and elegant and, as the preceding sections show, it is sufficient to represent the actual conditions at low temperatures. However, the prospects of carrying this treatment to a higher degree of approximation are not very favorable. The mathematical simplicity is lost and the theory becomes rather cumbersome because one has to increase the accuracy in so many different directions, For determining the Curie point one should, perhaps, try a different approach to the problem, for instance, that recently outlined by L. Brillouin[11] although it seems that this method, too, is difficult to be carried through without considerable idealizations.

[1] R. I. Allen and F. W. Constant, *Phys. Rev.*, **44**, 228 (1933).

[2] P. S. Epstein, *Phys. Rev.*, **41**, 91 (1932).

[3] P. Weiss, *Phys. Zeits.*, **9**, 361 (1908).

[4] W. Heisenberg, *Zeits. Physik*, **49**, 619 (1928).

[5] F. Bloch, *Ibid*, **61**, 206 (1930).

[6] R. Forrer, *Journ. Phys. et le Rad.*, **1**, 49 (1930).

[7] F. Bitter, *Phys. Rev.*, **37**, 91 (1931).

[8] A. Preuss, Thesis of Zurich (cited by Forrer).

[9] We infer this from the fact that our formula (8) with a suitably chosen constant represents well the saturation curves obtained by Weiss and his pupils for heterocrystalline materials.

[10] The formula given by Bloch (note 5) leads to a still much slower decline than ours.

[11] L. Brillouin, *Journ. Phys. et le Rad.*, **3**, 373, 565 (1932); **4**, 1 (1933).

A DUALITY IN INTERPOLATION TO ANALYTIC FUNCTIONS BY RATIONAL FUNCTIONS

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The following examples make clear a certain duality in interpolation to analytic functions by rational functions; reversal of the rôles of prescribed poles and points of interpolation reverses the region or regions of convergence, provided the bounding curves remain unchanged.

Ia. If the function $f(z)$ is analytic for $|z| \leqq R$, then the sequence of

rational functions of respective degrees n with poles at infinity (i. e., *sequence of polynomials in z of degree n*) *and which interpolate to f(z) in the origin, converges uniformly to f(z) for* $|z| \leqq R$.

Ib. If the function f(z) is analytic for $|z| \geqq R$, *then the sequence of rational functions of respective degrees n with poles at the origin* (i. e., *sequence of polynomials in* $1/z$ *of degree n*) *and which interpolate to f(z) at infinity, converges uniformly to f(z) for* $|z| \geqq R$.

IIa. If the function f(z) is analytic for $|p(z)| \leqq \mu > 0$, $p(z) = (z - z_1)(z - z_2)\ldots(z - z_\nu)$, *then the sequence of rational functions of respective degrees* $\nu n - 1$ *with poles at infinity* (i. e., *sequence of polynomials in z of degree* $\nu n - 1$) *and which interpolate to f(z) in the points* z_k *each considered of multiplicity n, converges uniformly to f(z) for* $|p(z)| \leqq \mu$.

IIb. If the function f(z) is analytic for $|p(z)| \geqq \mu > 0$, $p(z) = (z - z_1)(z - z_2)\ldots(z - z_\nu)$, *then the sequence of rational functions of respective degrees* νn *with poles in the points* z_k *each of multiplicity n and which interpolate to f(z) at infinity, converges uniformly to f(z) for* $|p(z)| \geqq \mu$.

IIIa. If the function f(z) is analytic for $|z| \leqq R$, $A < R < B$, *then the sequence of rational functions of respective degrees n with poles in the n points* $(B^n)^{1/n}$ *and which interpolate to f(z) in the* $n + 1$ *points* $(A^{n+1})^{1/(n+1)}$, *converges uniformly to f(z) for* $|z| \leqq R$.

IIIb. If the function f(z) is analytic for $|z| \geqq R$, $A < R < B$, *then the sequence of rational functions of respective degrees n with poles in the n points* $(A^n)^{1/n}$ *and which interpolate to f(z) in the* $n + 1$ *points* $(B^{n+1})^{1/(n+1)}$, *converges uniformly to f(z) for* $|z| \geqq R$.

IVa. If the function f(z) is analytic for $1/R \leqq |z| \leqq R > 1$, *then the sequence of rational functions of respective degrees* $2n$ *whose poles lie in the origin and the point at infinity each of multiplicity n and which interpolate to f(z) in the* $(2n + 1)$*st roots of unity, converges to f(z) for* $1/R \leqq |z| \leqq R$.

IVb. If the function f(z) is analytic for $|z| \leqq 1/R$ *and for* $|z| \geqq R > 1$, *then the sequence of rational functions of respective degrees* $2n - 1$ *whose poles lie in the* $(2n - 1)$*st roots of unity and which interpolate to f(z) in the origin and point at infinity each of multiplicity n, converges to f(z) for* $|z| \leqq 1/R$ *and for* $|z| \geqq R$.

In each of these theorems we are dealing with the extended plane, i.e., with the plane closed by the adjunction of the point at infinity. A rational function of degree n is a function of the form

$$\frac{a_0 z^n + a_1 z^{n-1} + \ldots + a_n}{b_0 z^n + b_1 z^{n-1} + \ldots + b_n},$$

where the denominator does not vanish identically.

All of these theorems are included in the general theorem proved below. Theorems I*a* and I*b* are classical. Theorem II*a* is well known and due to Jacobi. Theorem II*b* is new, and is remarkable in that *the region of con-*

vergence of the sequence of rational functions need not be simply connected, whereas the sequence is defined by interpolation in a single point. Theorem II*b* extends readily to the case of an arbitrary infinite region bounded by a finite number of Jordan curves, and the rational functions of interpolation can be defined for all degrees. Theorems III*a* and III*b* were recently established by the present writer,[1] as was also Theorem IV*a*.[2] Theorem IV*b* is new, but is similar to a theorem recently proved by Ketchum.[3] The theorem of Ketchum, like Theorem IV*b*, involves rational functions whose poles lie in roots of unity and which converge for $|z| \leqq 1/R$ and $|z| \geqq R > 1$ to an arbitrary function analytic for those values of z; the sequence of Ketchum does not seem, however, to be defined by interpolation.

Our general theorem is the following:

THEOREM. *Suppose the relation*

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{(z-\beta_{1n})(z-\beta_{2n})\ldots(z-\beta_{n+1,n})}{(z-\alpha_{1n})(z-\alpha_{2n})\ldots(z-\alpha_{n,n})}\right|} = \Phi(z) \not\equiv 0,\ \beta_{in} \neq \alpha_{jn}, \quad (1)$$

to be valid uniformly in some region D. Suppose $C: \Phi(z) = R$ and $\Gamma: \Phi(z) = R_1 > R$ are two curves or finite sets of curves in D such that $f(z)$ is analytic in a closed region or set of closed regions D_1 bounded by Γ and containing none of the points α_{in}, all the points β_{in}, and containing a closed region or set of closed regions D_2 bounded by C. Then the sequence of rational functions $r_n(z)$ of respective degrees n with poles in the points α_{in} which interpolate to $f(z)$ in the points β_{in} converges to $f(z)$ uniformly in D_2.

In (1), the fraction on the left is intended to represent an analytic function of degree $n + 1$ whose zeros are the points β_{in} and whose poles are the points α_{in} plus the point at infinity. In case a point α_{in} or β_{in} is at infinity, this notation requires some formal modification, and the same is true of the equations below.

$$f(z) - r_n(z) = \frac{1}{2\pi i}\int_\Gamma \frac{(z-\beta_{1n})(z-\beta_{2n})\ldots(z-\beta_{n+1,n})(t-\alpha_{1n})(t-\alpha_{2n})\ldots(t-\alpha_{n,n})f(t)dt}{(z-\alpha_{1n})(z-\alpha_{2n})\ldots(z-\alpha_{n,n})(t-\beta_{1n})(t-\beta_{2n})\ldots(t-\beta_{n+1,n})(t-z)}, \quad (2)$$

for z in D_2. The integrand (for z on C and t on Γ) may be split into three factors, one of which behaves in modulus like R^n, one of which behaves in modulus like $1/R_1^n$ and the third of which is uniformly bounded. Consequently the left-hand member approaches zero uniformly for z on C, hence uniformly for z in D_2, and the theorem is proved.

Under broad restrictions on the function $\Phi(z)$, it is true that *if* $f(z)$

is analytic on the closed set D_1, then the sequence $r_n(z)$ converges to $f(z)$ uniformly on that closed set.

Equation (1), valid uniformly on the curves C and Γ, implies the condition

$$\lim_{n\to\infty} \sqrt[n]{\left|\frac{(z-\alpha_{1n})(z-\alpha_{2n})\dots(z-\alpha_{n+1,n})}{(z-\beta_{1n})(z-\beta_{2n})\dots(z-\beta_{n,n})}\right|} = \frac{1}{\Phi(z)} \not\equiv 0 \qquad (3)$$

on C and Γ provided the points $\alpha_{n+1,n}$ and $\beta_{n+1,n}$ satisfy certain simple restrictions. Equation (3) is then sufficient for the application of the theorem just established, with the rôles of C and Γ, and of the α_{in} and β_{in} interchanged.

The points α_{in} and β_{in} need not be defined for every n.

Condition (1) is of interest in connection with the condition

$$\lim_{n\to\infty} \sqrt[n+1]{|(z-\beta_{1n})(z-\beta_{2n})\dots(z-\beta_{n+1,n})|} = \Psi(z) \qquad (4)$$

which has recently been emphasized by the present writer[4] as a useful and natural condition in connection with interpolation to analytic functions by polynomials. Many well-known and frequently utilized sets of points β_{in} satisfy condition (4) and hence can be used in connection with (1); compare Theorems I*a*–IV*b*. A similar remark holds for the points α_{in}.

Equation (1) may clearly be utilized to give results on the *degree of convergence* of the sequence $r_n(z)$ to the given function $f(z)$.

The duality that we have pointed out in connection with our main theorem is by no means dependent on condition (1). Let us consider the formal expansion of the particular function $f(z) = 1/(t-z)$ as a sequence of rational functions $r_n(z)$ found by interpolation in the points β_{in} and with poles in the points α_{in}. We have

$$\frac{1}{t-z} - r_n(z) = \frac{(z-\beta_{1n})\dots(z-\beta_{n+1,n})(t-\alpha_{1n})\dots(t-\alpha_{n,n})}{(z-\alpha_{1n})\dots(z-\alpha_{n,n})(t-\beta_{1n})\dots(t-\beta_{n+1,n})(t-z)}, \qquad (5)$$

and equation (2) is found formally by multiplying (5) through by $f(t)dt/(2\pi i)$ and integrating over Γ. The new formal expansion is justified if the right-hand member of (5) approaches zero uniformly for t on Γ with suitable restrictions on z, $f(z)$ and the β_{in}. Equation (5) corresponds, if z is considered variable, to interpolation to the function $1/(t-z)$ in the points β_{in} by a rational function of degree n with poles in the points α_{in}; but if t is considered variable, equation (5) corresponds to interpolation to the function $1/(t-z)$ in the points α_{in} (and infinity counted twice) by a rational function of degree $n+1$ with poles in the points β_{in}. It frequently occurs in practice that this exceptional interpolation at infinity can be replaced by interpolation in two suitably chosen additional points

$\alpha_{n+1,n}$ and $\alpha_{n+2,n}$. Under such circumstances, *if the right-hand member of* (5) *approaches zero uniformly for t on a curve or set of curves Γ and z on a curve or set of curves C, if Γ bounds a closed region or set of closed regions D_1 containing none of the points α_{in}, all the points β_{in}, and containing a set of closed regions D_2 bounded by C, and if C bounds a closed region or set of closed regions D_1' containing none of the points β_{in}, all the points α_{in}, and containing a set of closed regions D_2' bounded by Γ, then the sequence of functions $r_n(z)$ of respective degrees n with poles in the points α_{in} found by interpolation in the points β_{in} to a function $f(z)$ analytic in D_1 converges to $f(z)$ in D_2; the sequence of functions $r_{n+1}(z)$ of respective degrees $n+1$ with poles in the points β_{in} found by interpolation in the points α_{in} to a function $f(z)$ analytic in D_1' converges to $f(z)$ in D_2'.* The proof may be given directly from (5) and (2). The general theorem involving condition (1) is a special case, but one which is in rather more manageable form.

It is not unusual for the development of an analytic function $f(z)$ in a series of polynomials $p_n(z)$ (for example, Legendre polynomials) to be derived by contour integration from the development

$$\frac{1}{t-z} = \sum p_n(z)q_n(t);$$

it frequently occurs that a function $f(z)$ analytic in a region or set of regions can be developed in terms of the $p_n(z)$, and that a function $\phi(t)$ analytic on the complementary set can be developed in terms of the $q_n(t)$. Nevertheless, the remark we have made relative to *interpolation* seems to be new, and subject to many applications other than those we have given here. In particular, our remark applies to such series as

$$\sum \frac{a_n}{(z-\alpha_1)(z-\alpha_2)\ldots(z-\alpha_n)}, \qquad \sum b_n(z-\beta_1)(z-\beta_2)\ldots(z-\beta_n).$$

The former series corresponds to a sequence of rational functions of respective degrees n with poles in the points $\alpha_1, \alpha_2, \ldots, \alpha_n$ found from $f(z)$ by interpolation at infinity; the latter series corresponds to a sequence of rational functions of respective degrees n with poles at infinity found from $f(z)$ by interpolation in the points $\beta_1, \beta_2, \ldots, \beta_{n+1}$.

[1] Walsh, J. L., *Trans. Amer. Math. Soc.*, **34**, 22–74, 65 (1932).

[2] Walsh, J. L., *Proc. Nat. Acad. Sci.*, **19**, 203–207, Theorem III (1933).

[3] Ketchum, P. W., *Bull. Amer. Math. Soc.*, **39**, p. 347 (Abstract) (1933).

[4] Walsh, J. L., *Proc. Nat. Acad. Sci.*, **19**, 959–963 (1933).

GROUPS INVOLVING A SMALL NUMBER OF SQUARES

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A recent development with respect to groups of a finite order is the determination of all the groups which separately involve a given small number of operators which are squares thereunder. The special case when all the operators of a group have the same square presents no difficulties since the group is then abelian, of order 2^m, and of type $(1, 1, 1, \ldots)$. The fact that a group is necessarily abelian when all of its operators have a common square was noted by A. Cayley in what may be regarded as the first article on abstract groups of finite order, where he noted that if the product of two operators of order 2 is of order 2 then these two operators must be commutative.[1] The cases when no more than three of the operators of a group are squares thereunder were considered in these PROCEEDINGS, **19**, 848–851 (1933).

While the case when a group involves only one operator which is a square of an operator of the group is so simple as to appear trivial the case when a group involves two and only two operators which are squares thereunder presents several interesting features and yet is sufficiently elementary to justify the listing of these groups with those categories which are defined by simple abstract conditions. There are three infinite systems of such groups and every such group belongs to one and only one of these three systems. The first of these systems can be constructed by starting with the abelian group of order 2^m and of type $(1, 1, 1, \ldots)$, the second by starting with the abelian group of the same order but of type $(2, 1, 1, \ldots)$, and the third by starting with the Hamiltonian group of this order. In each system we can construct such a group of order 2^{m+1} containing this group of order 2^m provided the central of the latter contains more than one subgroup of order 2 and hence it is the direct product of a group of order 2^{m-1} and a group of order 2. The group of order 2^{m+1} in question can then be obtained by extending this group of order 2^m by an operator of order 2 which is commutative with every operator in the given group of order 2^{m-1} and transforms the given group of order 2 into itself multiplied by the operator of order 2 which is a square under the resulting group as well as under the given group of order 2^m when this is not abelian and of type $(1, 1, 1, \ldots)$. Whenever the centrals of the groups thus obtained involve more then one operator of order 2 the given construction can be repeated so as to obtain groups of order 2^{m+2} which satisfy the condition that each of them involves two and only two operators which are squares thereunder. This process may be repeated

until the centrals of the groups thus obtained involve only one operator of order 2. In each case exactly half of the additional operators are of order 2 while the rest are of order 4.

When a group involves a given number of operators which are squares then the direct product of this group and the abelian group of order 2^m and of type (1, 1, 1, ...) has the same property. Hence the given number of the operators which are squares cannot limit the number of the groups which satisfy this condition unless some other restriction is imposed. Whenever the product of two squares of such a group is not always a square then we can limit the number of groups which involve a given fixed number of squares by limiting the order of the centrals. For instance, there is only a finite number of groups which separately involve three and only three operators which are squares thereunder and whose central has an order which does not exceed a fixed number. When this fixed number is unity there is only one such group. When it is two there are two such groups and when it is three there are three such groups, etc. On the other hand, when a group involves two and only two operators which are squares thereunder the order of its central is at least 2, and there are exactly two such groups of order 2^m whenever m is odd and greater than unity while there is no such group when m is even.

If a group contains four and only four operators which are squares thereunder and these operators do not constitute a subgroup they generate a subgroup whose order is either 6 or 8 and the group is non-abelian. In the former case there are only two such groups when the central is of order 2, and every other such group is the direct product of one of these groups and the abelian group of order 2^m and of type (1, 1, 1, ...). One of these two groups is the dicyclic group of order 12 while the other is of order 24. When the four operators which are squares generate the subgroup of order 8 this subgroup appears in the central of the group, for if it did not appear in this central the group would involve a subgroup of order 6 which would include this subgroup of order 8 and whose operators would have three distinct squares. The squares of order 2 in the group could not be transformed into each other by an operator of this order since their product is not a square. As they could not be transformed into each other by an operator of order 4 in the group there results the following theorem: *If a group involves four and only four operators which are squares thereunder and if these four operators generate a subgroup of order 8 then this subgroup is contained in the central of the group.*

By means of this theorem it is easy to prove that there is one and only one group of order 32 which satisfies the condition that it contains four and only four operators which are squares thereunder and that these operators generate a subgroup of order 8. Moreover, every other group which satisfies this condition includes this group of order 32 and hence

its order cannot exceed 64 unless it is the direct product of such a group whose order divides 64 and an abelian group of the type (1, 1, 1, ...). It therefore results that the determination of all the groups which separately involve four and only four operators which are squares thereunder presents no serious difficulties except when these four operators constitute a subgroup of the group. In this case the number, of the possible distinct groups cannot be limited by imposing the condition that the order of the central does not exceed four. This number is, however, limited by the condition that the order of the central is 2.

When a group involves five and only five operators which are squares thereunder these five operators constitute a subgroup only when the group is of order five, of order 10 or the direct product of such a group and the abelian group of order 2^m and of type (1, 1, 1, ...). If these five operators include operators of order 3 they generate the non-cyclic abelian group of order 12 which is included in the central of the group. There is one such group of order 24 and every other such group whose order is divisible by 3 is the direct product of this group and the abelian group of order 2^m and of type (1, 1, 1, ...). In all other cases the five operators which are squares generate a subgroup of order 8, and the order of the group is a power of 2. These five operators are always relatively commutative in view of the theorem: *If a group involves less than nine operators which are squares thereunder then these squares are relatively commutative and if it involves less than three such operators they must appear in its central.*

When the five operators which are squares under a given group include two which are of order 4 the operators of order 8 found in the group generate a subgroup of index 2 which involves four and only four operators which are squares thereunder. This subgroup is either abelian or it involves a commutator subgroup of order 2, and it is a characteristic subgroup of the entire group since it includes all its operators of order 8 and is generated thereby. Each of the remaining operators of the group transforms into their inverse the operators of order 4 which are squares under the group, and hence each of these operators is either of order 2 or of order 4. There are two groups of order 2^m, $m > 6$, which involve separately five and only five operators which are squares thereunder including two which are of order 4. When $m = 6$ there is only one such group. If the order of the central of such a group exceeds 4 it is the direct product of such a group whose central is of order 4 and an abelian group of type (1, 1, 1, ...).

These results imply that the condition that a group contains only a small given number of squares is sometimes sufficient for the complete determination of all the groups satisfying this condition. While a given abstract number has not more than two distinct square roots there is no upper limit for the number of the distinct square roots of a group operator

in an arbitrary group. The determination of all the groups which involve a given number of powers greater than 2 presents much greater difficulties. Even the groups whose operators have a common cube have not yet been completely determined. The simplest general theorem relating thereto was proved by the late W. Burnside in 1902 and establishes the fact that all the operators which are conjugate under such a group are commutative. Somewhat later W. B. Fite published a number of additional properties of these groups.[2]

[1] A. Cayley, *Phil. Mag.*, **7**, 40–47 (1854).
[2] W. B. Fite, *Mathematische Annalen*, **67**, 498–510 (1909).

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*A THEORY FOR *-OPERATORS ANALOGOUS TO THE THEORY OF REDUCIBILITY FOR SELF-ADJOINT TRANSFORMATIONS IN HILBERT SPACE*

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The following conception is shown here to be useful in the theory of linear operations in Hilbert Space.

Definition.—A projection F is said to *break* the closed linear transformation A of domain $\mathfrak{D}$ everywhere dense in $\mathfrak{R}$ if (1) $Ff \epsilon \mathfrak{D}$ for all $f \epsilon \mathfrak{D}$, and (2) there exist a projection F' (independent of f) such that $AFf = F'Af$ for all $f \epsilon \mathfrak{D}$.

In the study of the general linear transformation L in n-dimensional unitary space, when the manifold $\mathfrak{M}$ reduces L, $\mathfrak{M}$ and its orthogonal complement are each transformed into themselves by L. Now if we merely require that the orthogonality of these two manifolds be preserved by L, we should be dealing with the n-dimensional version of breakage.

The transformations A and A^* being as above, we introduce with von Neumann[1] the associated transformations B and W. The following results are easy consequences of the theorems of von Neumann's paper.

THEOREM I. *The projection F breaks A, if and only if it reduces B. If F breaks A, then $F' = WFW^*$.*

THEOREM II. *Let A be as above. Then there exists a set of transformations $\{A_i\} i = 0, \pm 1, \pm 2, \ldots$, each closed and bounded with a bounded inverse, with domain $\mathfrak{D}_i$ and range R_i such that $\mathfrak{D}_i \perp \mathfrak{D}_j$ and $R_i \perp R_j$ for $i \neq j$ and such that*

$$f = Ef \pm \sum_{-\infty}^{\infty} f_i,\ f_i \in \mathfrak{D}_i,\ Af = O \cdot Ef + \sum_{-\infty}^{\infty} A_i f_i$$

where E is the projection on the manifold of zeros of A.

In the proof we take any set of numbers $\{\alpha_i\}$ such that $i = 0, \pm 1, \pm 2, \ldots, \alpha_i > \alpha_j$ for $i > j$ and $\lim_{i \to -\infty} \alpha_i = 0$ $\lim_{i \to +\infty} \alpha_i = \infty$, and letting $F_i = E(\alpha_i) - E(\alpha_{i-1})$, where $E(\lambda)$ is the resolution of the identity, corresponding to B, we take $A_i = A$, on range of F_i.

Since, if F reduces A, it breaks A (for then we have $F = F' = WFW^*$), we have the following,

THEOREM III. *F reduces A, if and only if it reduces B and W*.*

In the thesis which the writer will submit, these results are obtained in the case of a transformation between two spaces. Some further discussion is required.

[1] von Neumann, *Ann. Math.*, (2), **33**, 294–310 (1932).

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